

Theoretical and numerical investigation on impact noise radiated by collision of two cylinders[†]

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Abstract

Impact noise characterized by high peak value and short duration is very common source of noise in the industries and in many cases can be summed up in the noise radiated by collision of two cylinders. In order to precisely predict the impact noise radiated by collision of two cylinders, a modified theoretical prediction model was established based on the Palmgren's cylinder contact empirical model and acoustics theory. Then a numerical simulation method combining the finite element method (FEM) and the transient boundary element method (TBEM) had been presented to verify the modified theoretical model and further discuss the mechanism of impact noise. Both the contact force and impact noise of collision cylinders by the modified theoretical model were compared with the numerical results as well as the prediction results by the original theoretical model. It indicated that the results by modified theoretical prediction model are in good agreement with the numerical results, indicating better prediction superior to the original model. The impact noise radiated by collision cylinders is attributed to the rigid body acceleration. Furthermore, the experimental validations were conducted to verify the modified theoretical prediction model and numerical simulation results.

Keywords: Cylinder collision; Cylinder contact model; Impact noise; FEM/TBEM; Experimental validations

1. Introduction

Impact noise characterized by high peak value and short duration is very common source of noise in the industries. The mechanical impact phenomena can happen in riveting, forging, printing and gear teeth [1-3]. Although the contribution of acoustic energy is negligible in mentioned energy transforms, the sensitivity of humane auditory system forces designers to attend the noise radiated from their designs. The countries of the world have established safety standards for impact noise based on experimental studies. Several investigators have reported on the impact noise radiated by collision of two spheres by using the analytical, numerical and experimental method [4-6]. Zheng et al. [7] presented a practical approach for predicting the meshing noise due to the impact of chain rollers against the sprocket of chain drives. An acoustical model relating dynamic response of rollers and its induced sound pressure was developed based on the fact that the acoustic field was mainly created by oscillating rigid cylindrical rollers. Kadmiri et al. [8] investigated the rattle noise of automotive gearboxes, resulting from impacts between

toothed wheels of unselected gear ratios with experimental and numerical analysis. Pinte et al. [9, 10] discussed the effectiveness of an active structural acoustic control (ASAC) system for the reduction of repetitive impact noise.

Mechanical impact noise in many cases can be summed up in noise radiated by collision of two cylinders, such as revolute joint with spatial clearance and gear mesh impact noise. Yufang and Zhongfang [11] studied the impact of cylinders on the basis of the Hertz contact model and acoustic theory and the contact impact force and acoustic pressure had been proposed. However, the cylinder contact model had been simplified into sphere contact model and the line contact condition had been substituted by point contact in their analysis. Consequently, the original theoretical prediction model proposed by Yufang and Zhongfang should be modified based on the cylinder contact force model to precisely estimate the contact impact force and acoustic pressure.

As we all know, the Hertz contact theory remains the foundation for almost all of the available contact force models. However, Hertz contact theory is restricted to the evaluation of the contact stiffness parameter and nonlinear exponent, particularly when the bodies contact in a line or surface instead of a point [12-14]. In the interest of consummating contact theory, several compliant cylinder contact force models

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have been proposed over the last decades [15-17]. Nevertheless, these contact force models are nonlinear implicit function and require an iterative solution scheme such as the Newton–Raphson method, to solve for the normal contact force. In addition, the current cylindrical contact models include logarithmic functions, which impose mathematical and physical limitations on their application. Lankarani and Nikraves [18] proposed a cylinder contact force model based on the Hertz model where a term is added to account for the energy dissipation that occurs in the impact phenomenon. The proposed model is an explicit function of the contact force and no iterative method is required to obtain the contact force in each time of simulation. However, the nonlinear power exponent n in the model ranges from 1~1.5 extending in this form the use of the original Hertz model to handle cylindrical contact. Harris [19] proposed an explicit line-contact empirical model of cylinder contact, namely the Palmgren’s cylinder contact model and showed that the formula is effective to solve the cylinder contact problem.

In order to precisely predict the impact noise radiated by collision of two cylinders, a modified theoretical prediction model of collision cylinders is established based on the Palmgren’s cylinder contact empirical model and acoustics theory. Then a numerical simulation method combining the finite element method (FEM) and the transient boundary element method (TBEM) has been presented to verify the modified theoretical model and further discuss the impact noise mechanism. In the first step of numerical method, collision of cylinders is simulated and then the state variables produced in previous step are imported into transient boundary element model. It indicates that the results by the modified model are in good agreement with the numerical results, indicating better prediction superior to the original model. The impact noise radiated by collision cylinders is attributed to the rigid body acceleration. Finally, the experimental validations are conducted to verify the modified theoretical prediction model and numerical simulation results.

2. Theoretical model

2.1 Contact-impact theory

On the basis of contact dynamics and the Palmgren’s cylinder contact empirical model [19], the nonlinear function between contact impact force and indentation can be expressed as

$$F = K\delta^n \quad (1)$$

where F is the contact impact force, δ is the relative indentation, n is the nonlinear power exponent, K is the contact stiffness parameter. According to the Palmgren’s cylinder contact empirical model, the nonlinear power exponent $n = 10/9$ and the contact stiffness parameter K which depends on the contact length and is independent of the contact radii of the bodies is given by

$$K = \frac{1}{1.36^n} E^* L^{\left(\frac{8}{9}\right)} \quad (2)$$

where L is the contact length, E^* represents the composite modulus of collision cylinders and is evaluated as

$$\frac{1}{E^*} = \frac{1-\nu_i^2}{E_i} + \frac{1-\nu_j^2}{E_j} \quad (3)$$

where the quantities E_i, ν_i and E_j, ν_j are the elastic modulus and Poisson coefficients of collision cylinders, respectively.

On the basis of Newton’s law of motion and Palmgren’s cylinder contact model, the largest relative indentation δ_m and contact force F_m as well as contact time t_c can be obtained by

$$\delta_m = \left(\frac{n+1}{2} \frac{m_{red}}{K} v_0^2 \right)^{\left(\frac{1}{n+1}\right)} \quad (4)$$

$$F_m = K \left(\frac{n+1}{2} \frac{m_{red}}{K} v_0^2 \right)^{\left(\frac{n}{n+1}\right)} \quad (5)$$

$$t_c = \frac{2\delta_m}{v_0} \int_0^1 \frac{d(\delta/\delta_m)}{\sqrt{1-(\delta/\delta_m)^{(1+n)}}} \quad (6)$$

where m_{red} represents equivalent mass, as shown below.

$$m_{red} = \frac{m_i m_j}{m_i + m_j} \quad (7)$$

where m_i and m_j are the mass of colliding cylinders, respectively.

Assuming that contact force and impact acceleration time history is approximately half wave sine pulse, then the contact force and impact acceleration of collision cylinders can be written in the form:

$$\begin{aligned} F(t) &= F_m \sin \omega_c t \\ a(t) &= a_m \sin \omega_c t \end{aligned} \quad (8)$$

where ω_c is angular frequency which is defined as $\omega_c = \pi/t_c$.

2.2 Acoustic analysis

The acoustic equation for the cylinder in a cylindrical coordinate system [7, 11] is given by

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} &= \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} \\ p &= \rho \frac{\partial \Phi}{\partial t} \\ u_r &= -\frac{\partial \Phi}{\partial r} \end{aligned} \quad (9)$$

where Φ is the velocity potential, c is the sound speed in the

air, t is time, p is the sound pressure and u_r is the particle velocity.

If the cylinder vibrates with a velocity of $v_0 e^{j\omega t}$, the velocity potential $\Phi(r, \theta, t)$ becomes

$$\Phi(r, \theta, t) = A_1 \cos \theta H_1^{(2)}(kr) e^{j\omega t} \quad (10)$$

where $H_1^{(2)}(kr)$ is the first order Hankel function of the second kind, ω is the angular frequency of oscillation and k is equal to ω/c . If the plane of vibration is taken as the reference plane, the angle between the velocity of the cylinder surface and the plane of vibration is θ , then with the boundary condition

$$(u_r)_{r=R} = -\frac{\partial \Phi}{\partial r} \Big|_{r=R} = V_0 e^{j\omega t} \cos \theta \quad (11)$$

where R is the radius of the cylinder. Coefficient A_1 in Eq. (10) can be therefore obtained as

$$A_1 = -\frac{V_0}{kH_1^{(2)'}(kR)} \quad (12)$$

where $H_1^{(2)'}(kR)$ is the differentiation of $H_1^{(2)}(kr)$ with respect to kr at $r = R$.

$$\Phi(r, \theta, t) = -\frac{V_0 \cos \theta}{kH_1^{(2)'}(kR)} H_1^{(2)}(kr) e^{j\omega t} \quad (13)$$

The velocity potential for any arbitrary velocity of the oscillating cylinder can be obtained by Fourier synthesis of this solution when the Fourier transform of the velocity is known. The velocity potential for an arbitrary $V_0(\omega)$ is

$$\Phi(r, \theta, t) = -\frac{\cos \theta}{2\pi} \int_{-\infty}^{\infty} \frac{V_0(\omega) H_1^{(2)}(kr) e^{j\omega t}}{kH_1^{(2)'}(kR)} d\omega \quad (14)$$

The velocity of the cylinder due to a unit impulse of acceleration can be expressed as

$$V_0(\omega) = B\delta(\omega) + \frac{1}{j\omega} \quad (15)$$

where B is an arbitrary constant and $\delta(\omega)$ is the Delta function. The velocity potential due to a unit impulsive acceleration then becomes

$$I(r, \theta, t) = -\frac{\cos \theta}{2\pi} \int_{-\infty}^{\infty} \frac{B\delta(\omega) + \frac{1}{j\omega}}{kH_1^{(2)'}(kR)} H_1^{(2)}(kr) e^{j\omega t} d\omega \quad (16)$$

The velocity potential from the acceleration $a(t)$ of the oscillating cylinder is

$$\Phi(r, \theta, t) = -\frac{\cos \theta}{2\pi} \times \int_{-\infty}^{\infty} a(\tau) \frac{B\delta(\omega) + \frac{1}{j\omega}}{\left(\frac{\omega}{c}\right) H_1^{(2)'}\left(\frac{\omega R}{c}\right)} H_1^{(2)}\left(\frac{\omega r}{c}\right) e^{j\omega(t-\tau)} d\omega d\tau \quad (17)$$

For the far sound field condition, the first order Hankel function of the second kind and its derivative can be approximately simplified as

$$H_1^{(2)}(z) = \sqrt{\frac{2}{\pi z}} e^{-j\left(z - \frac{3}{4}\pi\right)} \left[1 + \frac{3}{8jz} + \frac{15}{128z^2}\right] \quad (18)$$

$$H_1^{(2)'}(z) = \sqrt{\frac{2}{\pi z}} e^{-j\left(z - \frac{1}{4}\pi\right)} \left[1 + \frac{7}{8jz} - \frac{57}{128z^2}\right] \quad (19)$$

Substituting Eqs. (18) and (19) into Eq. (16) and then we can obtain the velocity potential from a unit impulse of acceleration as follows:

$$I(r, \theta, t) = -\frac{\cos \theta}{2\pi} j c \sqrt{\frac{R}{r}} \times \int_{-\infty}^{\infty} \frac{\left[B\delta(\omega) + \frac{1}{j\omega}\right] \left[1 + \frac{3c}{8j\omega r} + \frac{15c^2}{128(\omega r)^2}\right] e^{j\omega\left(t - \frac{r-R}{c}\right)}}{\omega \left[1 + \frac{7c}{8j\omega r} - \frac{57c^2}{128(\omega r)^2}\right]} d\omega \quad (20)$$

On the basis of contour integration in the complex plan, Eq. (20) is calculable as

$$I(r, \theta, t) = \frac{16}{19} \sqrt{\frac{R}{r}} \cos \theta \frac{R^2}{r} \times \left\{ e^{-l_2 t'} \left[\frac{1}{\sqrt{65}} \left(\frac{19r}{R} - 7 \right) \sin l_1 t' - \cos l_1 t' \right] + 1 \right\} \quad (21)$$

where

$$l_1 = \frac{\sqrt{65}c}{16R}, l_2 = \frac{7c}{16R}, t' = t - \frac{r-R}{c} \quad (22)$$

Substituting Eq. (21) into Eq. (17) and then we can obtain the velocity potential

$$\Phi(r, \theta, t) = \frac{16}{19} \sqrt{\frac{R}{r}} \cos \theta \frac{R^2}{r} \times \int_{-\infty}^{\infty} a(t-\tau) \left\{ e^{-l_2 t'} \left[\frac{1}{\sqrt{65}} \left(\frac{19r}{R} - 7 \right) \sin l_1 t' - \cos l_1 t' \right] + 1 \right\} d\tau \quad (23)$$

The sound pressure caused by the impact acceleration of a cylinder is given by

$$p(r, \theta, t) = \rho \frac{\partial \Phi(r, \theta, t)}{\partial t}. \quad (24)$$

By means of superposition theorem, the total sound pressure radiated by collision of two cylinders can be calculated

$$p_{red}(r, \theta, t) = p_i(r_i, \theta_i, t) + p_j(r_j, \theta_j, t). \quad (25)$$

Thus, we have established the modified theoretical prediction model of collision cylinders based on the Palmgren cylinder contact empirical model and acoustics theory.

3. Numerical simulation

To verify the modified theoretical model and further discuss the mechanism of impact noise radiated by collision of two cylinders, the FEM/TBEM has been presented to calculate the contact force and transient impact noise.

3.1 Simulation of impact

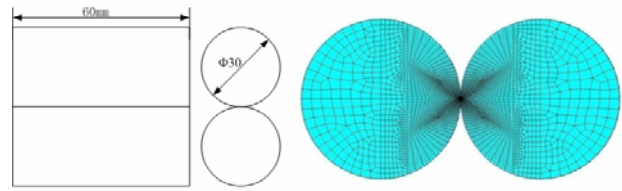
Almost in all of acoustical-structure interaction, it is assumed that the effect of air on structure is negligible when a structure is interacted with air because of the difference between the elastic waves speed in air and in the solids. In this simulation, the effect of air on cylinder is ignored so that the problem is simulated uncoupled. Therefore, the dynamical differential equation of cylinder contact-impact can be expressed as

$$M_s \ddot{U} + C_s \dot{U} + K_s U = F_s \quad (26)$$

where M_s is mass matrix, C_s is damping matrix, K_s is stiffness matrix and F_s is external load.

For simplicity in this equation, all other loading terms except the surface traction have been neglected. In addition, it will be seen that acoustic waves are propagated in less than one millisecond duration of time and the damping is ignored because of this short duration of time and substance of the cylinders.

The structure model of collision cylinders is shown in Fig. 1(a). The length of cylinders is 60 mm and the radius is 15 mm. The material parameters of the cylinders are chosen as follows: the density $\rho_i = \rho_j = 7850 \text{ kg/m}^3$, the elastic modulus $E_i = E_j = 206 \text{ GPa}$, the Poisson coefficients $\nu_i = \nu_j = 0.3$. The finite element model of collision cylinders is shown in Fig. 1(b). In order to simulate the contact impact of the cylinders, the structure model is dispersed into 113230 elements by hexahedral solid element with 8 nodes. Moreover, the two cylinders are applied with initial relative impact velocity of 0.5 m/s respectively in consideration that the impact noise radiated by collision cylinders is mostly dependent on the surface accelerations and the contact force depends on the relative impact velocity rather than the absolute impact velocity.



(a) The structure of cylinders (b) Finite element of cylinders

Fig. 1. Models of collision cylinders.

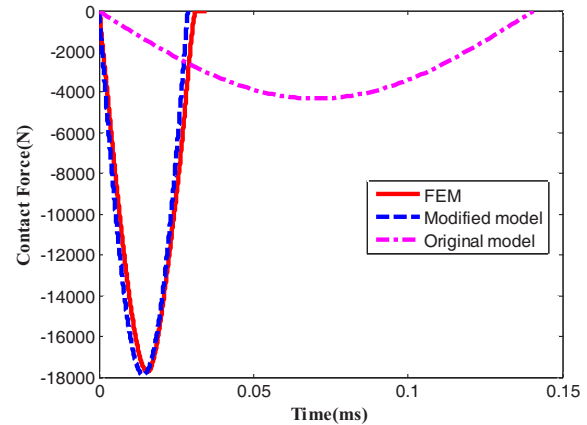


Fig. 2. Time history of contact force.

In addition, the contact impact process consists of contact constraint; therefore it needs to calculate contact properties in each increment. The contact force calculated by the modified theoretical model is compared with the numerical results by the finite element method (FEM) as well as the original theoretical prediction model proposed by Yufang and Zhongfang [11]. Yufang and Zhongfang deduces following equations to calculate duration of contact and the maximum contact force of collision cylinders.

$$t_c = \frac{2\delta_m}{v_0} \int_0^{\delta_m} \frac{d(\delta/\delta_m)}{\sqrt{1 - (\delta/\delta_m)^2}} \quad (27)$$

$$F_m = K \left(\frac{5}{4} \frac{m_{red}}{K} v_0^2 \right)^{\frac{3}{5}}. \quad (28)$$

In these equations, v_0 is impact velocity, K is the effective contact stiffness, m_{red} is effective mass. The effective contact stiffness and effective mass are calculated as follows:

$$K = \frac{4}{3} \frac{1}{1 - \nu_i^2 + \frac{1 - \nu_j^2}{E_j} \sqrt{\frac{1}{R_i} + \frac{1}{R_j}}} \quad (29)$$

$$m_{red} = \frac{m_i m_j}{m_i + m_j} \quad (30)$$

where the quantities E_i , ν_i and E_j , ν_j are the elastic modulus and

Poisson coefficients of collision cylinders, respectively, m_i and m_j are the mass of colliding cylinders, respectively.

By assuming half-sine behavior for contact force, the time history of contact force is shown in Fig. 2. Solid line was taken from the FEM numerical results; dash-line was taken from the modified theoretical prediction model and dash dot line was taken from the original theoretical prediction model.

We can see that the numerical result of contact force is approximately half wave sine pulse, verifying the half-sine behavior assumption. The result by modified theoretical prediction model is reasonably consistent with the numerical result; whereas the contact force based on the original theoretical prediction model may cause greater errors. The reason is that the real line-contact model of collision cylinders has been simplified into the Hertz sphere contact model and the line contact condition has been substituted by the point contact in the original theoretical prediction model, resulting in the extension of contact time and reduction of contact force amplitude. It can be concluded that the modified theoretical prediction model indicates better prediction for the contact force of collision cylinders superior to the original model.

It is well known that the impact noise radiated by collision cylinders is mainly attributed to the impact acceleration. In order to analyze mechanism of impact noise radiated by collision of cylinders, the time history of acceleration and acceleration spectrum are plotted on Figs. 3(a) and (b), respectively. The acceleration curve is transformed by Fourier series to determine the spectrum.

The natural frequencies of cylinders are shown in Table 1. Based on the theory of impact dynamics and stress wave spreading, natural mode of vibration will be excited during collision of two cylinders. It is obvious that the picks of spectrum curve are merged on natural frequencies by comparing the acceleration spectrum with natural frequencies. Moreover, we can see from the acceleration spectrum that the frequencies of structure acoustic radiation are above audible frequency range (20 KHz), and the noises with these frequencies do not have any influences on the radiation of audible sounds. Therefore, noises with frequency above 20 KHz are wept out from Fig. 3 by using a low-pass filter and then the result of filtering is compared with the rigid body acceleration which is calculated by dividing the contact force to the mass of the cylinder. Comparative figure is shown in Fig. 4 and we can conclude that the impact noise radiated by collision cylinders is attributed to the rigid body acceleration because only frequency of rigid body motions is in audible range.

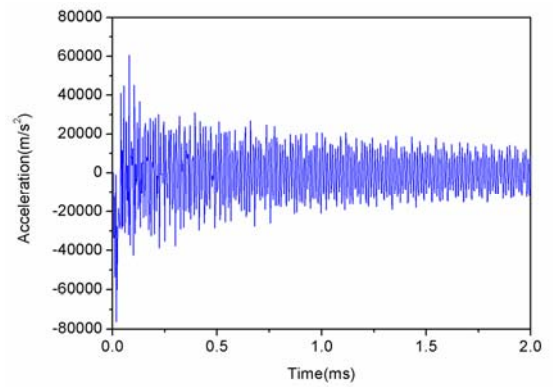
3.2 Acoustic simulation

In ideal fluid medium, acoustic wave equation [20] can be expressed as

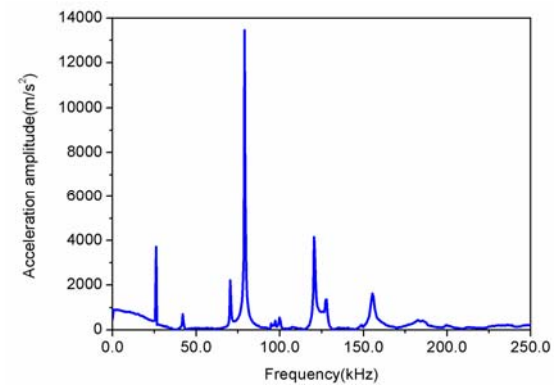
$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} . \tag{31}$$

Table 1. Nature frequency of cylinder.

Mode number	Nature frequency (Hz)
1	26159
2	26159
3	26559
4	42169
5	48398
6	48398
7	53120
8	70721
9	70721
10	71613



(a)



(b)

Fig. 3. (a) Time history of acceleration; (b) acceleration spectrum.

In this simulation, the transient boundary element method (TBEM) is used to solve Eq. (31) and simulate the transient impact noise of collision cylinders. The transient boundary element model of collision cylinders is shown in Fig. 5.

Import the vibration displacement of collision cylinders calculated in previous section into transient boundary element model, and then we can obtain the sound pressure contour of acoustic pressure field propagation as shown in Fig. 6. The visualization of transient sound wave has been realized. We can see that along with the propagation of sound wave, the

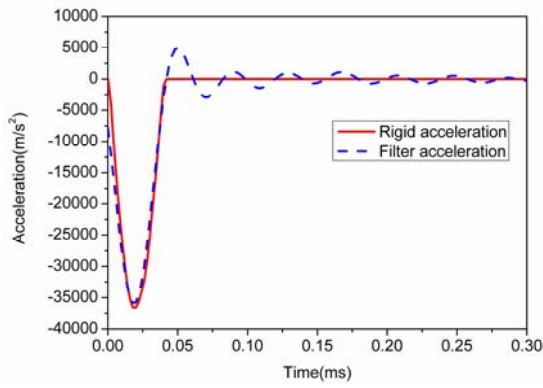


Fig. 4. Filtered acceleration and rigid acceleration.

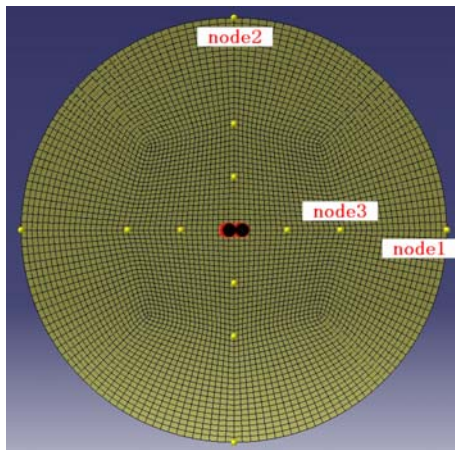


Fig. 5. Transient boundary element model of collision cylinders.

energy of impact noise diffuses and the acoustic pressure amplitude decreases gradually.

In order to validate the rationality and validity of the modified theoretical prediction model of collision cylinders, the time history of the radiated sound pressure for node 1 depicted in Fig. 5 is shown in Fig. 7. The solid line illustrates the TBEM numerical results, the dash line indicates the modified theoretical prediction model, and the dash dot line shows the original theoretical prediction model proposed by Yufang and Zhongfang [11]. The figure shows a good agreement between the modified model and the TBEM numerical result. Theoretical errors are created by the reflection of sound between the cylinders. In the superposition that is applied in theoretical method, the reflection of sound is ignored. The little error of the simulation possibly is occurred by inaccurate material properties that are used in the simulation.

4. Experimental validation

In order to further verify the modified theoretical prediction model and numerical simulation results, we perform the experimental measurements of the impact noise radiated by collision of two cylinders. We design two sets of experimental

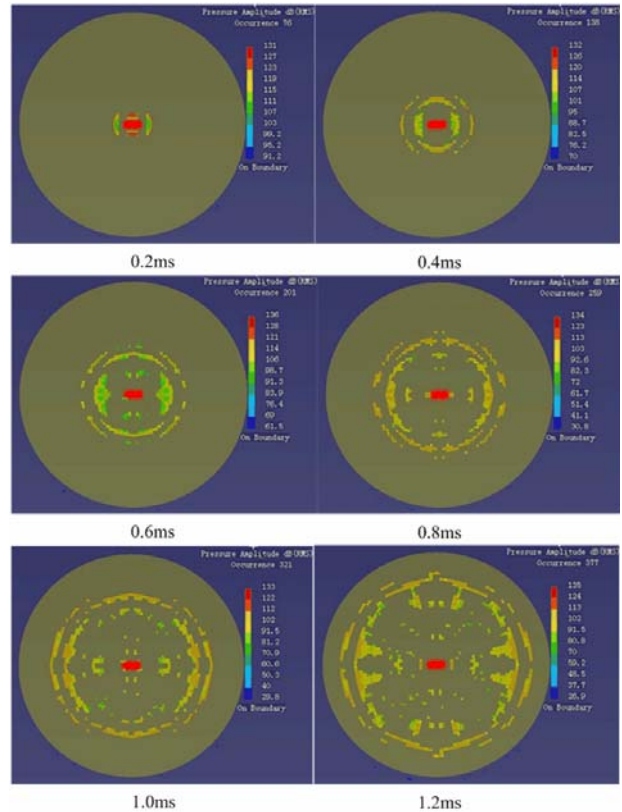


Fig. 6. Acoustic pressure field propagation.

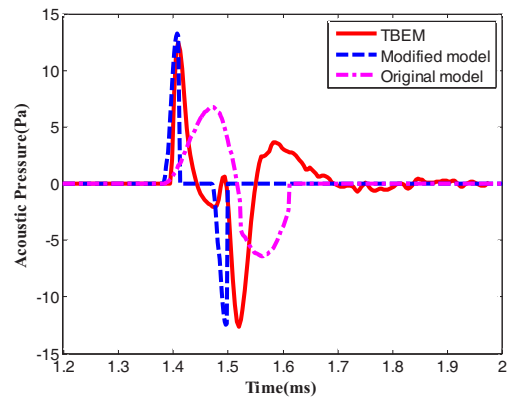


Fig. 7. Time history of sound pressure for node 1.

scheme to explain the difference between the modified theoretical model and the original theoretical model as shown in Fig. 8. The key difference between the two sets of experimental scheme is that the cylinders are hanged by soft ropes in Fig. 8(a) while by hard beams in Fig. 8(b). M+P test system and SmartOffice software (M+P international Germany) with two microphones is employed in the experiments. The length of the hanging ropes and beams (0.3 m) is large enough firstly to achieve impact velocities of several meters per second and secondly to make the circular trajectory of the cylinder approximates a rectilinear horizontal trajectory during impact.

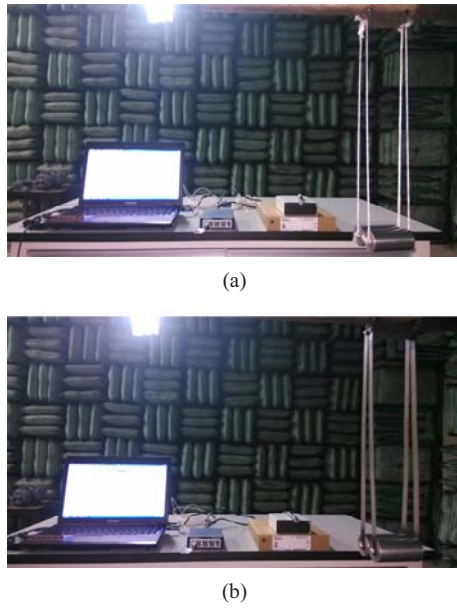


Fig. 8. Experimental schemes: (a) cylinders hanged by ropes; (b) cylinders hanged by beams.

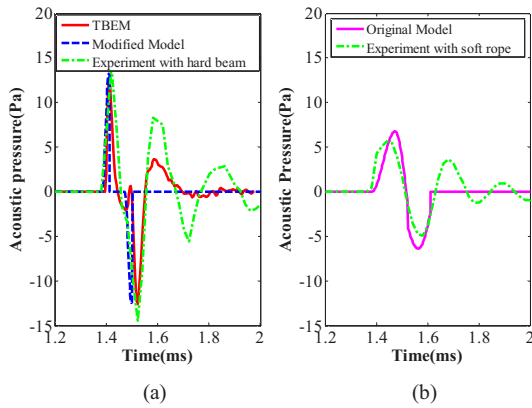


Fig. 9. Experimental validation of impact noise: (a) cylinders hanged by hard beams; (b) cylinders hanged by soft ropes.

The height at which the cylinder is manually released (with zero initial speed) determines its impact velocity on the other cylinder. In order to obtain 1.0 m/s initial impact velocity, we move one of the cylinders from upright position to the height at 0.05 m and release it with zero initial speed. The experiment results of impact noise with two different experimental schemes are compared with the modified theoretical model and numerical simulation results as shown in Fig. 9.

It has been found from experiment process that it is almost impossible to realize the line-contact of collision cylinders with soft ropes. We can see that the experiment result with hard beams satisfying the line-contact condition is consistent with the modified theoretical model and numerical simulation result as shown in Fig. 9(a). The main cause of error comes from the mass of hard beam and the damping of the system. On the other hand, we can find that the experiment result with

soft ropes attributed to the Hertz point contact is in good agreement with the original theoretical model as shown in Fig. 9(b).

5. Conclusion

In this paper, in order to precisely predict the impact noise radiated by collision of two cylinders, a modified theoretical prediction model has been established based on the Palmgren’s cylinder contact empirical model and acoustics theory. Then a numerical simulation method combining the finite element method and the transient boundary element method has been presented to verify the modified theoretical model and further discuss the mechanism of impact noise. The results show a good agreement between the modified theoretical predictions and numerical simulation results; whereas the original theoretical prediction model proposed by Yufang and Zhongfang [11] may cause greater errors. The modified theoretical prediction model can indicate better estimation superior to the original theoretical prediction model. The impact noise radiated by collision cylinders is attributed to the rigid body acceleration. Finally, the experimental validations are conducted to verify the modified theoretical prediction model and numerical results. It can be concluded that the modified theoretical prediction model could precisely the assessment of acoustic of impacting cylinders.

Acknowledgments

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Nomenclature

- F : Contact impact force
- δ : Relative indentation
- n : Nonlinear power exponent
- K : Contact stiffness
- L : Contact length
- E^* : Effective elasticity modulus
- E : Elastic modulus
- ν : Poisson coefficients
- ρ : Density
- R : Radius of cylinder
- m_{red} : Effective mass
- m : Mass of cylinder
- t_c : Contact time
- ω_c : Angular frequency
- Φ : Velocity potential
- c : Sound speed in the air
- t : Time
- p : Sound pressure
- u_r : Particle velocity
- θ : Degree from impact axes
- r : Distance from impact point

v_0	: Impact velocity
M_s	: Mass matrix
C_s	: Damping matrix
K_s	: Stiffness matrix
F_s	: External load

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