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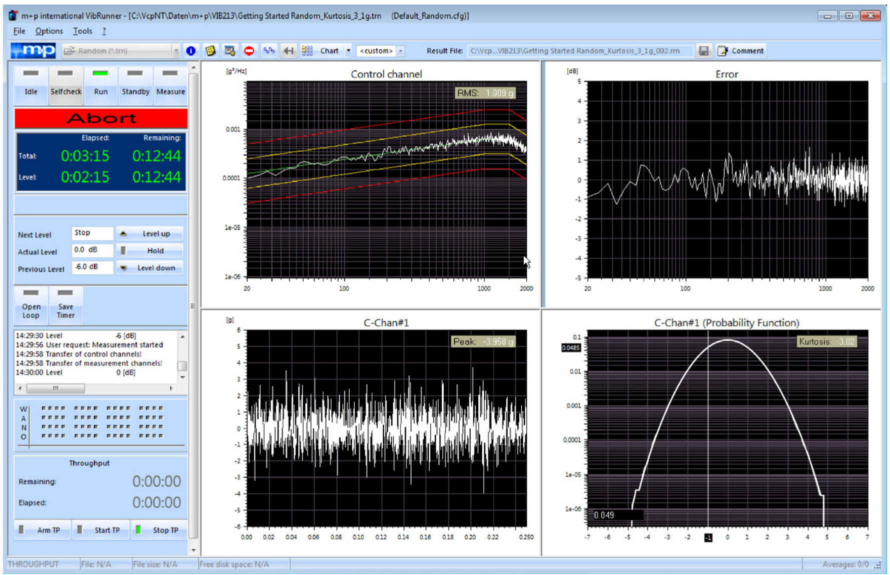
Application Note

The Effect of Kurtosis on Random Vibration Testing



- Simulating real world vibration environments better
- Increasing the higher peak events in a controlled way
- More effective than traditional Gaussian random testing
- m+p VibControl vibration control software
- m+p Analyzer for data capture and recording
- m+p VibPilot acquisition hardware

Introduction

'Any vibration is described by the time history of motion, where the amplitude of the motion is expressed in terms of displacement, velocity or acceleration. Sinusoidal vibration is the simplest motion,' as its deterministic, its frequency content and amplitude are defined, so 'the motion of vibration can be predicted at any point in time', due to its periodic nature. A pure sine waveform is composed of a signal frequency; more complex waveforms arise due to overlaying multiple sine waveforms of different frequencies at the same instance in time. Due to the deterministic nature of sinusoidal waveform it is often used to characterize the vibrational motion of structures. (1) (2)

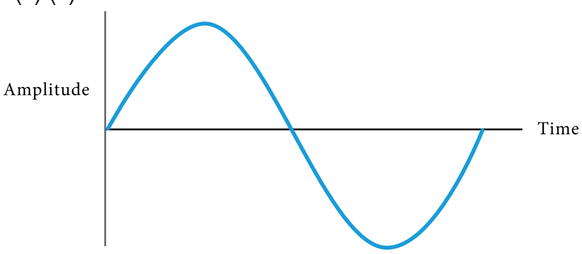


Figure.1a Sinusoidal waveform (1)

Figure.1b Random waveform (1)

Random vibration is characterised as non-deterministic motion, which is composed of a multitude of frequencies excited at the same time, as a result its future behaviour cannot be predicted based on its past behaviour (aperiodic nature). (1) Random vibration is recognized as a more realistic approach of simulating the effects vibrations have on objects/systems in the real world. Consider simulating the oscillating motions induced on a car stereo while driving. The stereo will be subjected to vibrating motion from the car engine, the road surface, the cars gears turning and etc.. None of the motions mentioned are deterministic, due to the fact road surfaces are uneven as you drive, and the speed of the car is never really constant, therefore the work rate of the engines and gears are not constant resulting in a non-deterministic motion. (1) (2)

'The instantaneous amplitude of a random vibration cannot be expressed mathematically as an exact function of time, it is possible to determine the probability of occurrence of a particular amplitude on a statistical basis' through the utilization of a Probability Density Function. This is the core ethos of generating random vibration in test procedures. (1) For any random vibration test for a given Probability Density Function based on user's specification, the waveform produced during testing will always be unique and can never be repeated for the same random test, further mimicking the real world.

Random vibration testing does not always capture all the essential effects of real world motions. Let's re-examine the excitation of a car stereo specifically the vibrations induced on the object as a result of road surface. As the vehicle is driving along the road a random vibration is induced on the car stereo, as shown in figure.3, at time T the car passes over a pothole resulting in a high amplitude event being subjected on the car stereo, as illustrated on the vibration waveform.

Figure.2 Car drives over pothole (4)

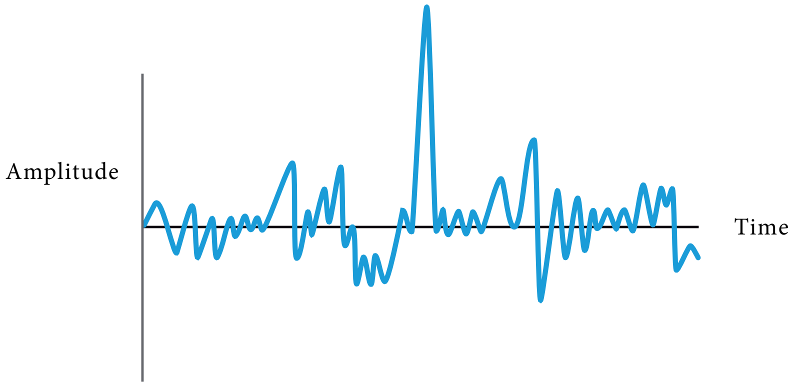


Figure.3 Peak amplitude event (1)

As mentioned before random vibrations are non-deterministic, and cannot be mathematically expressed as an equation like a sine wave. Probability Density Functions are applied to statistically predict the temporal occurrence of amplitude events, which are further utilized in software packages, such as m+p VibControl software to generate unique random vibration waveforms for testing. Unfortunately high amplitude events as described above are harder to recreate in test procedures, due to nominal Gaussian nature of the probability distribution. This is why m+p have introduced a feature to allow greater controllability of the distribution of Probability Density Functions, by altering its Kurtosis, which in turn affects the probability of occurrence of peak events in random vibrations.

Statistically Predicting Random Vibrations

There is no true mathematical equation to represent a random vibration, but the probability of the occurrence of amplitude events within a range of values can be statistically predicted, using a Probability Density Function. 'The most commonly used probability distribution is the normal (Gaussian) distribution' (1), which is expressed as:

$$p = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where:

- μ is the mean
- σ is the standard deviation
- σ^2 is the variance

Figure 4 shows a graphical representation of a Gaussian Probability Density Function.

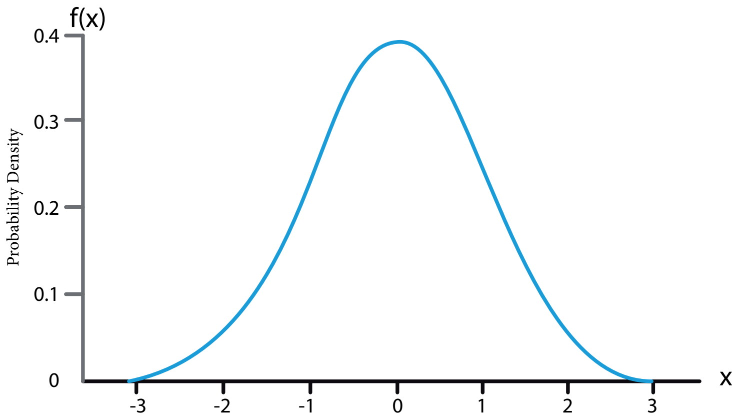


Figure.4 Gaussian (normal) Probability Density Function (1)

The probability of a value (in this case an amplitude) occurring between the range x_0 and x_1 is equivalent to the area under the curve between the two values. Mathematically the area under the curve between two points is solved by integration; therefore the probability of amplitude occurring between two points is as expressed as an integral of the function between the two points. This is depicted in figure.5 and written as a basic integration equation. (1) (2)

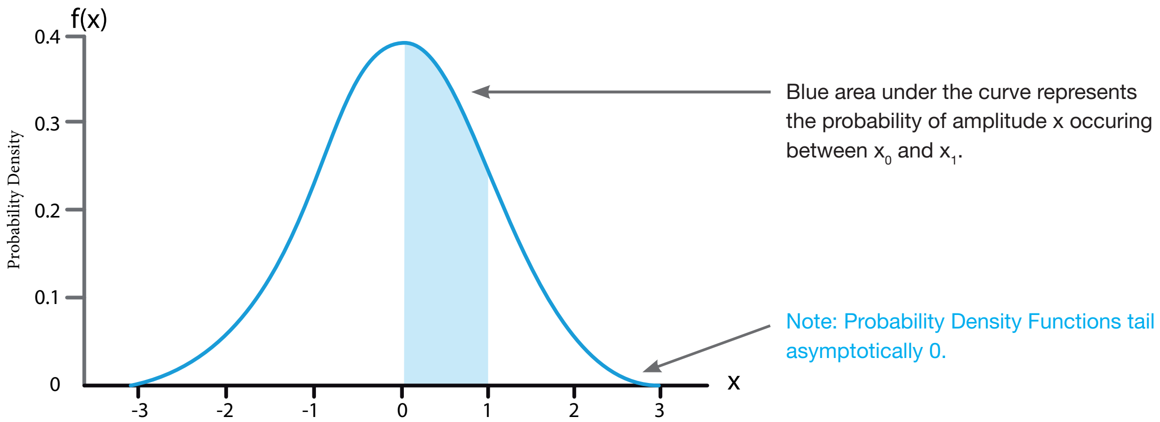


Figure.5 Calculating the probability between two points

$$P(x_1 > X > x_0) = \int_{x_0}^{x_1} f(x)dx$$

Skewness & Kurtosis

Skewness

There are two statistical values that describe the shape of a distribution, Skewness & Kurtosis. Skewness is measure of the symmetry of a distribution, which gives an insight in how data is distributed about the mean (i.e. the Skewness of the tails of the distribution). (3) It is mathematically expressed as:

$$a_3 = \sum \frac{(X_i - \overline{X})^3}{n\sigma^3}$$

Where:
- X_i is the i^{th} value of data
- $\overline{X}$ is the mean value
- n is the sample size
- σ is the standard deviation

For the purpose of this report assume the sample size is large, in order to reduce the complexity of the equation.

Note: Skewness α_3 'is referred to as the "third standardized central moment for the probability model'. (3)

Figure.6 illustrates negative Skewness (α_3 is negative value). Data is more distributed about the right of the mean (the left tail is bigger than the right). (3)

Figure.7 illustrates positive Skewness (α_3 is positive value). Data is more distributed about the left of the mean (the right tail is bigger than the left). (3)

Figure.8 illustrates symmetric Skewness (α_3 is approximately 0). Data is symmetrically distributed on either side of the mean (the right tail is symmetric with the left). Figure.8 represents a normal distribution. (3)

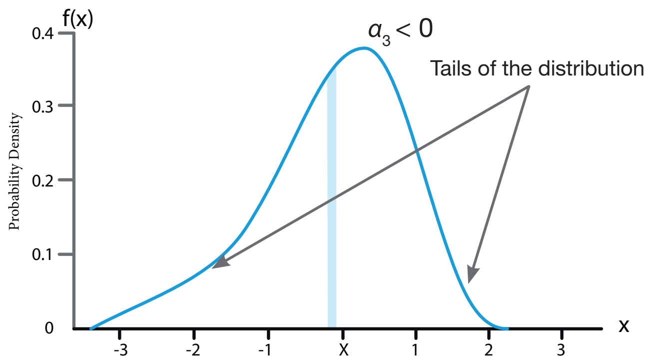


Figure.6 Negative Skewness

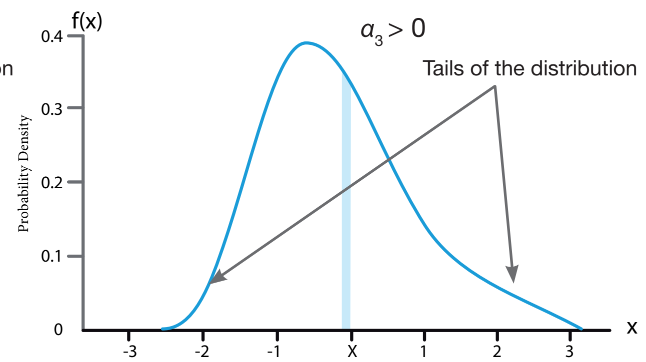


Figure.7 Positive Skewness

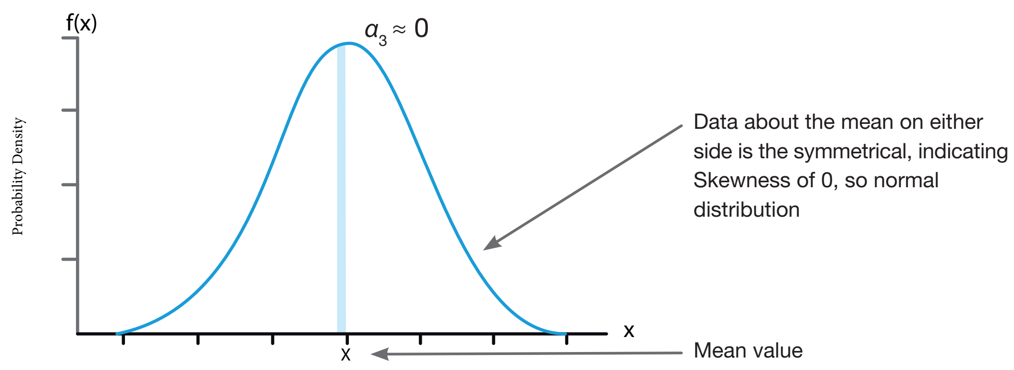


Figure.8 Symmetrical Skewness

Kurtosis

The second value that describes the shape of the distribution is the Kurtosis, a long held misconception has led many to define the Kurtosis as the degree of peakedness (or flatness) of a distribution. This is inaccurate as the Kurtosis 'measures the combined weight of the tails relative to the rest of a distribution'. (3)

Kurtosis is mathematically expressed as:

Where:
- X_i is the i[th] value of data
- $\overline{X}$ is the mean value
- n is the sample size
- σ is the standard deviation

$$a_4 = \sum \frac{(X_i - \overline{X})^4}{n\sigma^4}$$

For the purpose of this report assume the sample size is large, in order to reduce the complexity of the equation.

Note: Kurtosis α_4 'is referred to as the "fourth standardized central moment for the probability model'. (3)

A Gaussian or normal distribution has a Kurtosis equal to 3. In many software packages the equation below is used to define Kurtosis.

$$a_4 = \sum \frac{(X_i - \overline{X})^4}{n\sigma^4} - 3$$

For the purpose of this report assume the sample size is large, in order to reduce the complexity of the equation.

This equation subtracts 3 from the standard Kurtosis equation, thus many software packages recognize a normal distribution of having a Kurtosis of 0, which is commonly referred to as excess Kurtosis. (3) For the purposes of this report the non-excess Kurtosis principle will be utilized throughout the report. (m+p VibControl software sets the Kurtosis of Gaussian (normal) distribution to 3).

Within vibration testing the Gaussian (normal) distribution has been the set standard of defining a Probability Density Function, which is used to generate a random signal based on the statistical probability of an amplitude event occurring within a given range, as per example below:

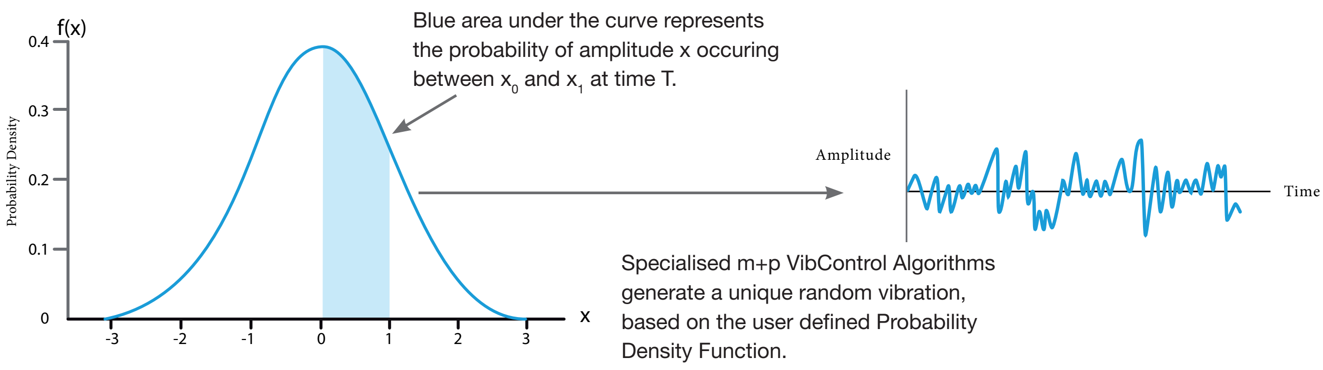


Figure.9 Generating a random vibration from a Probability Density Function

For vibration testing, increasing the value of Kurtosis increases the 'heaviness' of the tails of the amplitude distribution, which in turn increases the probability of peak events occurring in the generated random vibrations in comparison to a Gaussian Probability Density Function. (1) (3)

The diagram below illustrates the varying 'heaviness' of the tails of a distribution with different Kurtosis values. (Data was generated through the Kurtosis feature in m+p VibControl software for a 1 g rms signal and captured by m+p Analyzer program to generate the graph below). It is clearly illustrated on the plot that with increasing Kurtosis values the heaviness of the tails of the distribution increase as well as resulting in a greater coverage of extreme values in comparison to a normal distribution. The Probability Density curves also show well-conditioned behaviour with smoothly increasing probability for higher amplitudes with higher Kurtosis, illustrating the effectiveness and reliability of m+p VibControl software.

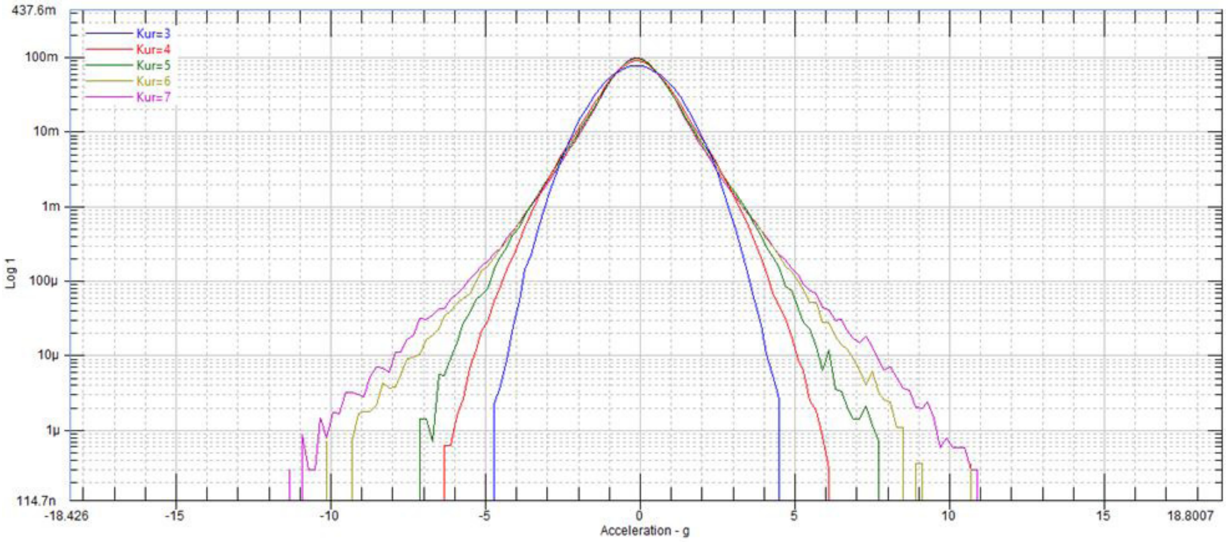
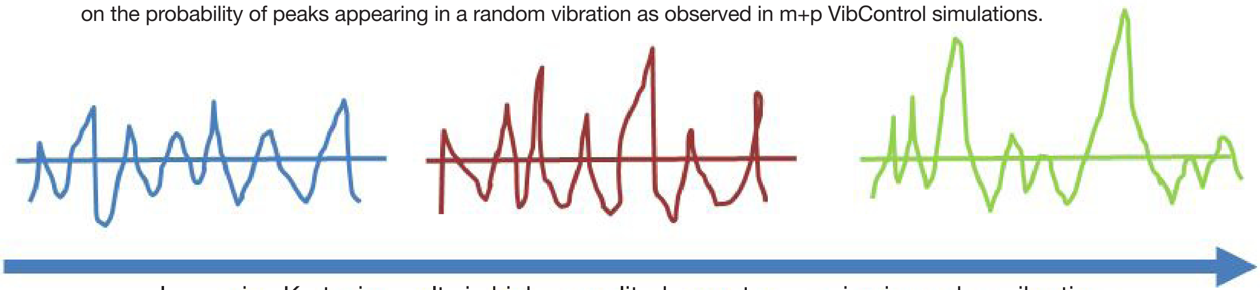


Figure.10 Varying Kurtosis values vs. distributions tail 'heaviness'

The diagram depicts the fundamental effect increasing Kurtosis of a Probability Density Function has on the probability of peaks appearing in a random vibration as observed in m+p VibControl simulations.



Increasing Kurtosis results in higher amplitude events occurring in random vibrations

Figure.11 Increasing Kurtosis values vs. peaks in random vibration

Crest Factor

The crest factor (cf) is a ratio between the rms value of a signal and its peak value, which is expressed as below:

$$Crest\ Factor = \frac{Peak\ Amplitude}{rms\ value}$$

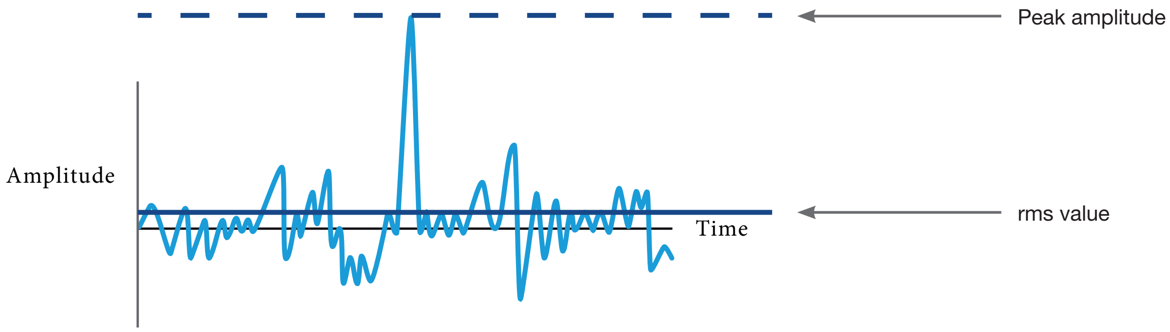


Figure.12 Diagram illustrating the crest factor of a random vibration

There is no direct relationship between the cf of a random signal and the Kurtosis of a Probability Density Function, but theoretically increasing the Kurtosis increases the probability of higher peak events occurring compared with a Gaussian distribution. Therefore in practice there is an increase in the crest factor of random signal with a higher Kurtosis value when compared with a Gaussian distribution. Because the intent of increased Kurtosis is to better simulate peak events there is no attempt in the m+p VibControl software to control crest factor when Kurtosis is set to > 3. However when Kurtosis is set to 3 the sigma clipping value controls the crest factor.

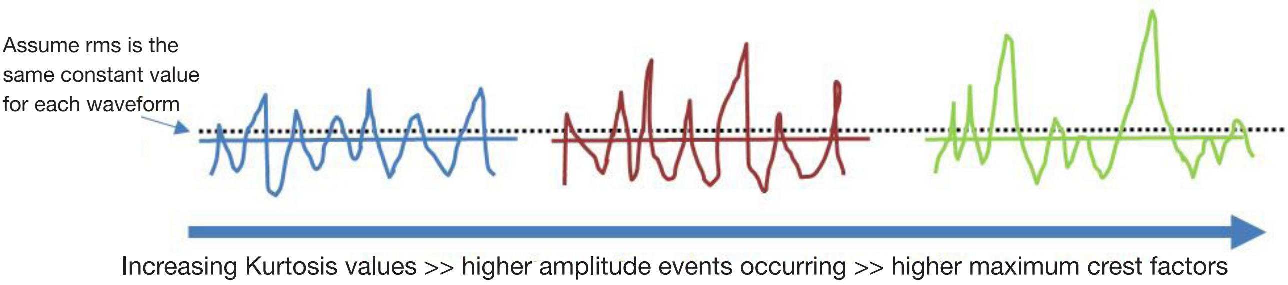


Figure.13 Diagram illustrating relationship of Kurtosis to crest factor.

RMS and Sigma Clipping

In m+p VibControl software the spectral content is defined using a table of Power Spectral Density (PSD) values from which the rms signal level is derived from the area under the PSD curve. The rms amplitude is defined as one sigma on the Probability Density curve. The characteristic shapes of the amplitude distribution are defined by user inputs of Kurtosis and sigma clipping. Three times the rms value is referred to as three sigma, etc.. The statistical mean value is set to zero for the purposes of signal generation.

A Kurtosis setting of 3 defines the traditional Gaussian distribution for random testing.

Defining the power spectral density of a random signal

No.	Frequency	Accel.
1	50 Hz	1.26e-004 g^2/Hz
2	1000 Hz	6.28e-004 g^2/Hz
3	1500 Hz	6.28e-004 g^2/Hz
4	2000 Hz	3.77e-004 g^2/Hz

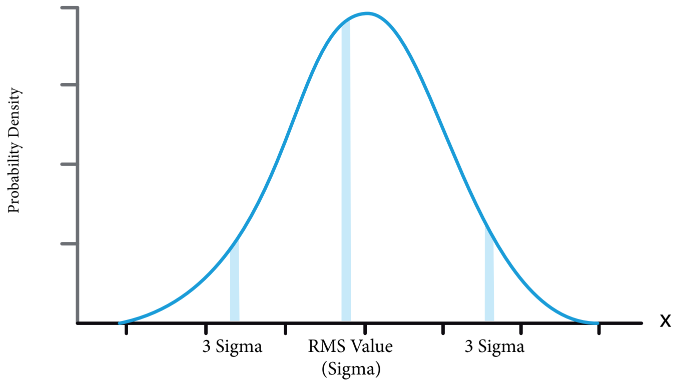
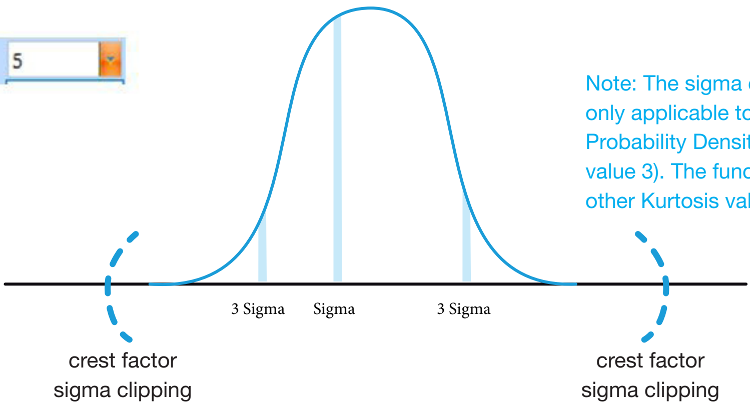


Figure.14 Defining the parameters of a random signal

Since Probability Density Functions tail to zero asymptotically (i.e. there could be infinite peaks but at low probability). This can lead to unwanted harmonics being generated in random vibrations, because some peaks will need clipping/limiting to accommodate the electronics and shaker systems-leading to erroneous testing. When a Gaussian distribution is being used then the m+p VibControl software incorporates sophisticated crest factor sigma clipping algorithms to eliminate the harmonic residue. In m+p VibControl crest factor sigma clipping controls the generated random signal to the peak value set by the user. If a sigma clipping of five is selected then the peak value generated in the random vibration is approximately 5 times the defined rms value. For example if a random signal with a defined rms of approximately 5 g is generated with a sigma clipping of 3, the peak value generated in the vibration during test is approximately 15 g (three times the rms value). The effectiveness of this sigma clipping control can be seen in figure.17a below.

Sigma clipping: 5



Note: The sigma clipping functionality is only applicable to Gaussian distributed Probability Density Functions (i.e. Kurtosis value 3). The functionality is disabled for all other Kurtosis values.

Figure.15 Sigma clipping in m+p VibControl

As discussed, setting Kurtosis to a value greater than 3 allows the Probability Density curve to be weighted towards higher values compared to the Gaussian case and hence higher crest factors are to be expected and desirable. In this case the m+p VibControl software no longer attempts to limit the crest factor. Since there is no definitive formula relating Kurtosis with crest factor the following section investigates the empirical relationship between them using the m+p VibControl software.

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Testing the Kurtosis Feature in m+p VibControl Software

A test was set up to investigate the effects changing the Kurtosis value of a random test profile has on the peak values generated in a random signal during a closed loop test. The crest factor of the positive peaks as well as the negative peaks is observed as well. The rms value of the for all the Kurtosis tests is set to approximately 1 g rms in m+p VibControl, shown in the test profile below.

Figure.16 Defining the parameters of the random signal in m+p VibControl's reference page

For each Kurtosis value 3 to 12 a closed loop test was run for approximately 15 minutes on one control channel on an m+p VibPilot. m+p Analyzer was used in conjunction with NI USB-9234 module to record the:

- Positive peak time history
- Negative peak time history
- Positive crest factor history = $\frac{Positive\ peak\ time\ history}{rms\ time\ history}$ Note: Calculated utilizing m+p Analyzer Calculator function
- Negative crest factor history = $\frac{Negative\ peak\ time\ history}{rms\ time\ history}$ Note: Calculated utilizing m+p Analyzer Calculator function

Kurtosis 3

Test 1

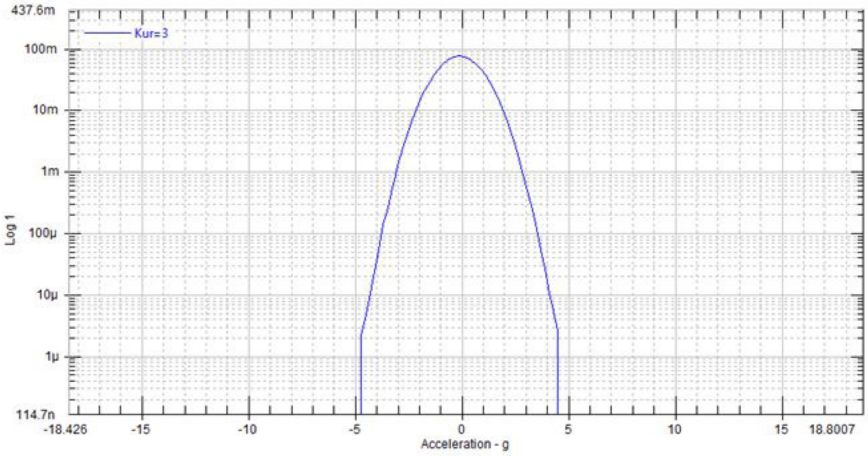

Figure.17a Probality Density Function for Kurtosis 3

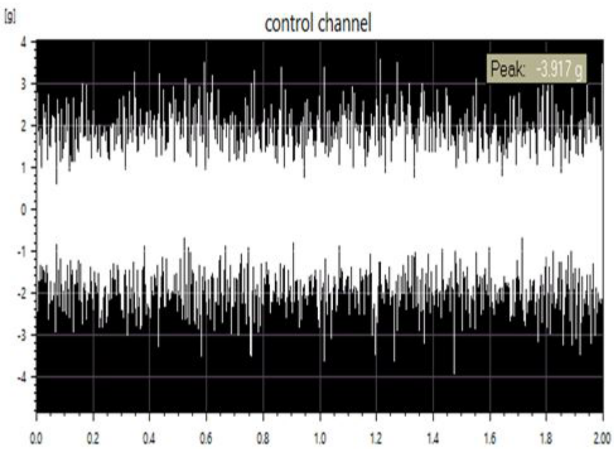

Figure.17b Random signal in time domain for Kurtosis 3

For the random signal with a Kurtosis of 3 (i.e. standard Gaussian), it is observable that on both positive and negative peak time history, there is a uniform distribution of peak events, with highest positive peak of 5.0007 g and negative peak of 4.853 g. It is important to point out the sigma clipping for this test profile was set to 5, thus the highest peak events that can occur in the random signal is approximately 5 g for 1 g rms signal. This is clearly the case as observed from the results, which demonstrates the power and effectiveness of m+p sigma clipping algorithms. The highest positive peak crest factor was 4.9059, while the negative was 4.8104.

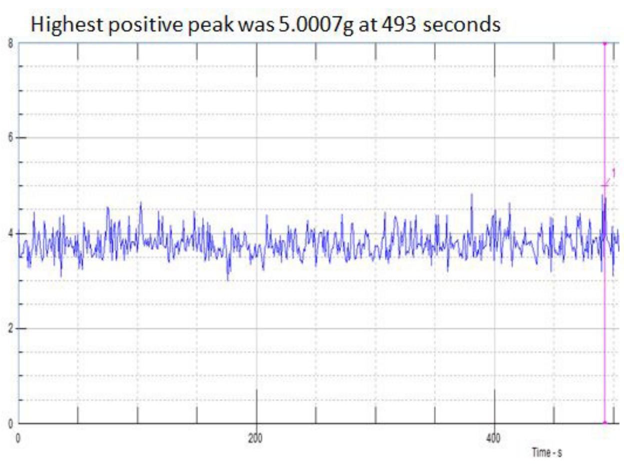

Figure.18 Positive peak time history for Kurtosis 3

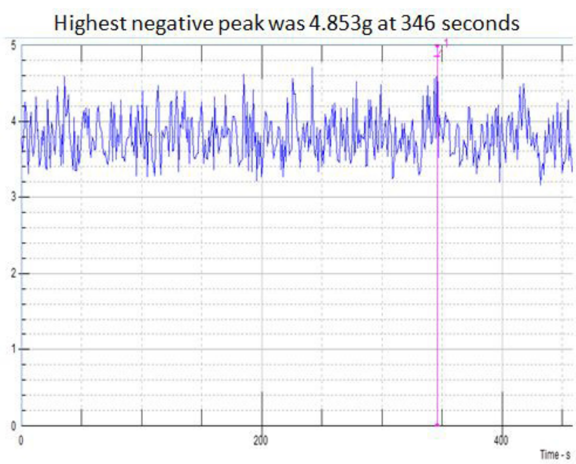

Figure.19 Negative peak time history for Kurtosis 3

Positive crest factor of 4.9059 at 493 seconds

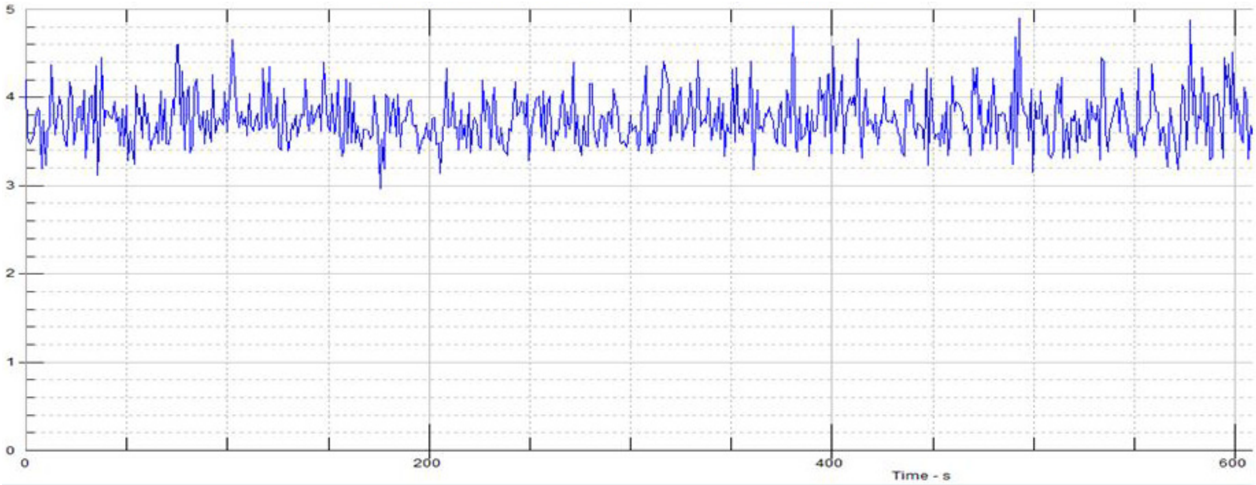

Figure.20 Positive crest factor time history for Kurtosis 3

Negative Crest factor of 4.8104 at 346 seconds

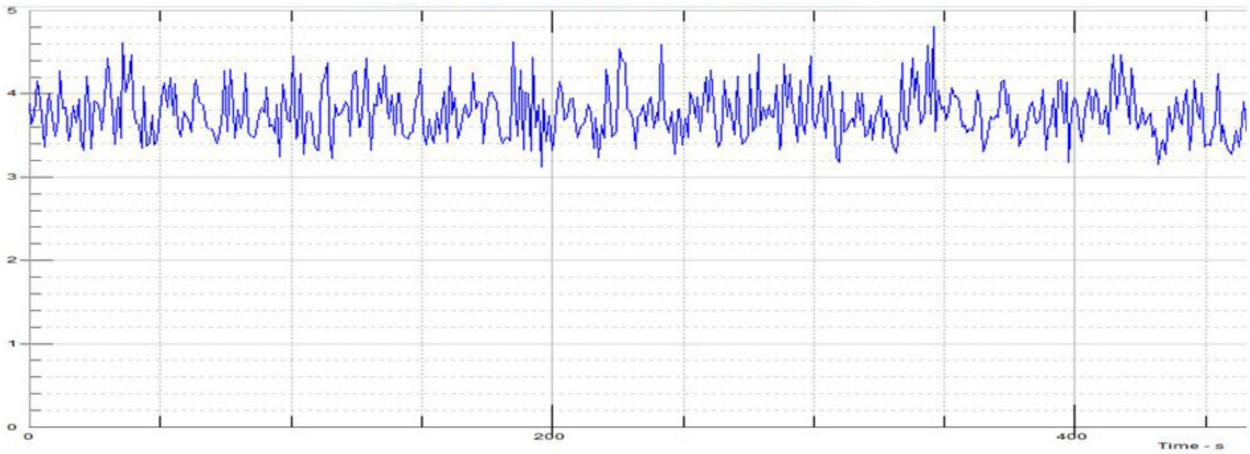

Figure.21 Negative crest factor time history for Kurtosis 3

Kurtosis 4

Test 1

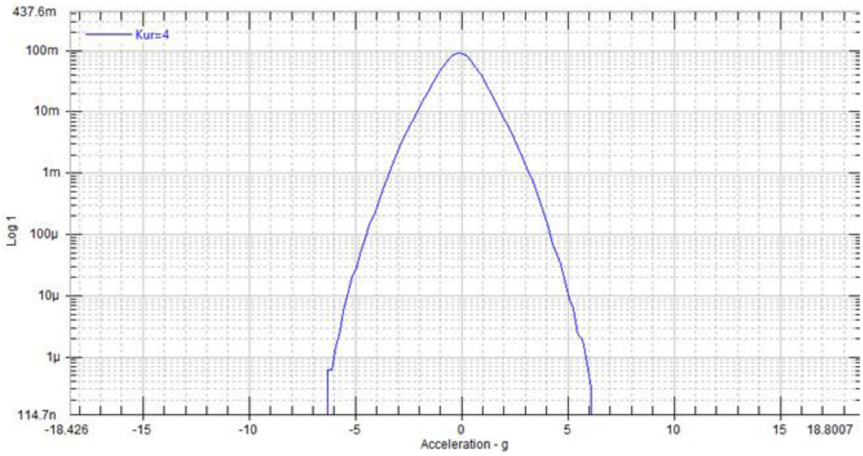

Figure.22a Probality Density Function for Kurtosis 4

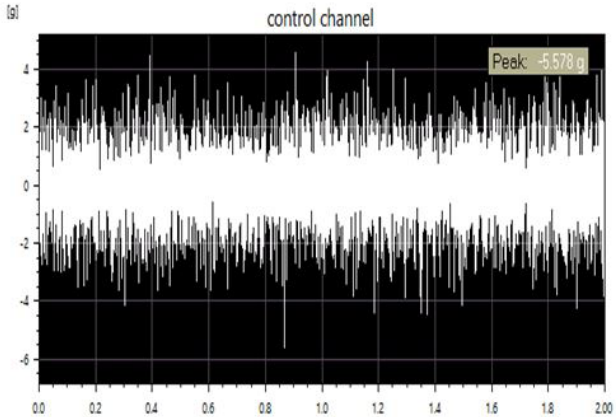

Figure.22b Random signal in time domain for Kurtosis 4

There was a higher occurrence of higher amplitude peaks when the Kurtosis was set to 4, in comparison to the nominal Gaussian distribution. The highest positive peak to occur was 7.5011 g and the negative peak was 6.8004 g. The respective crest factors for the amplitudes were 7.3211 and 6.6536.

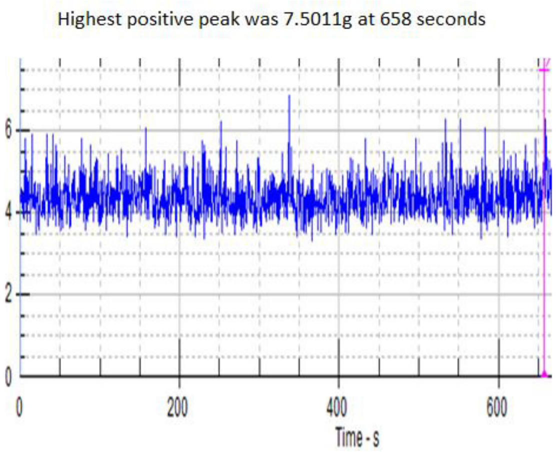

Figure.23 Positive peak time history for Kurtosis 4

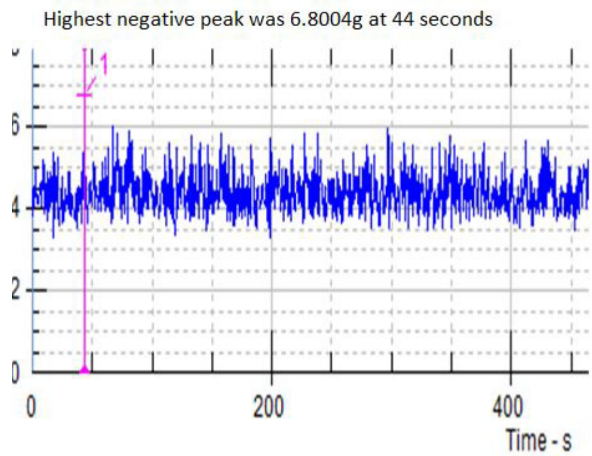

Figure.24 Negative peak time history for Kurtosis 4

Positive crest factor of 7.3211 at 658 seconds

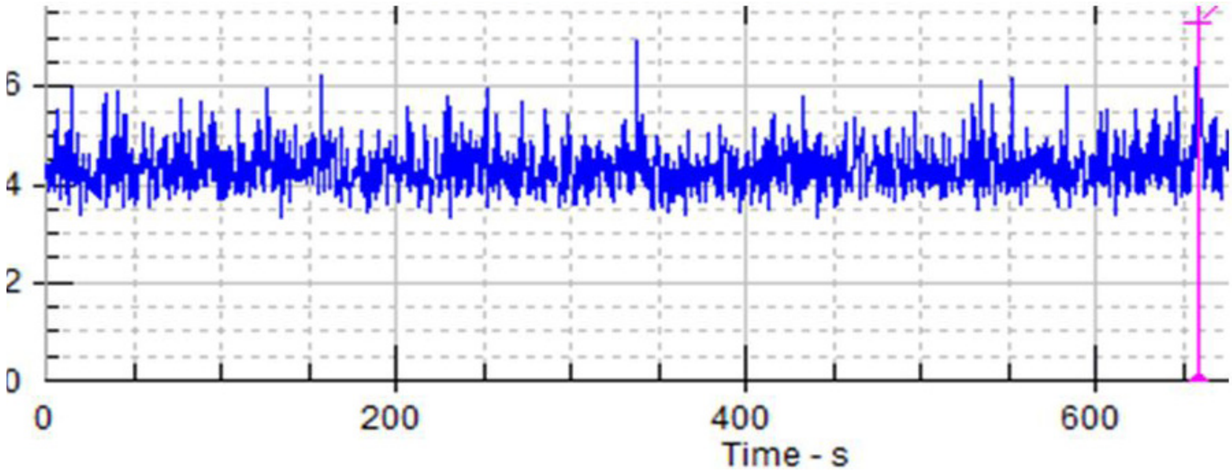

Figure.25 Positive crest factor time history for Kurtosis 4

Negative crest factor of 6.6536 at 44 seconds

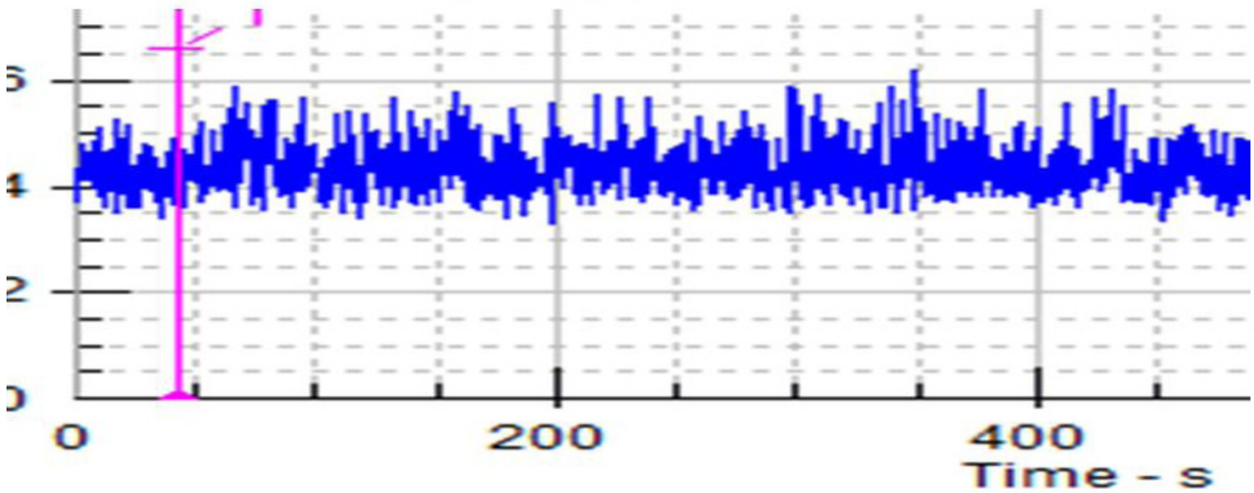

Figure.26 Negative crest factor time history for Kurtosis 4

Kurtosis 5

Test 1

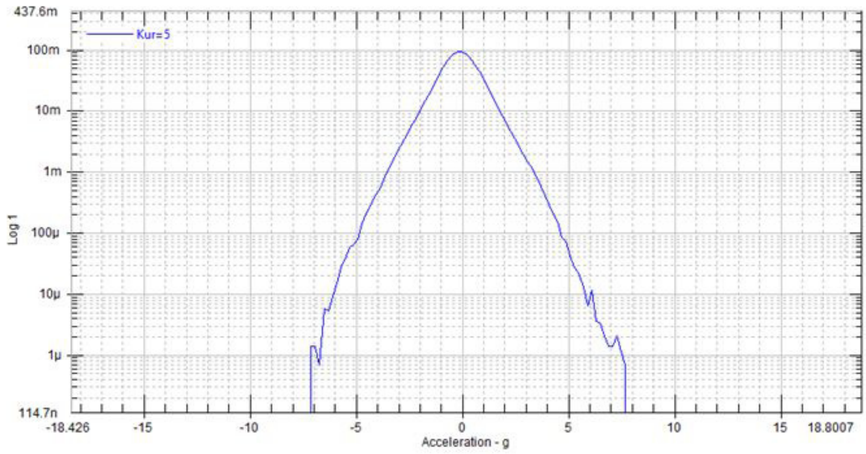

Figure.27a Probality Density Function for Kurtosis 5

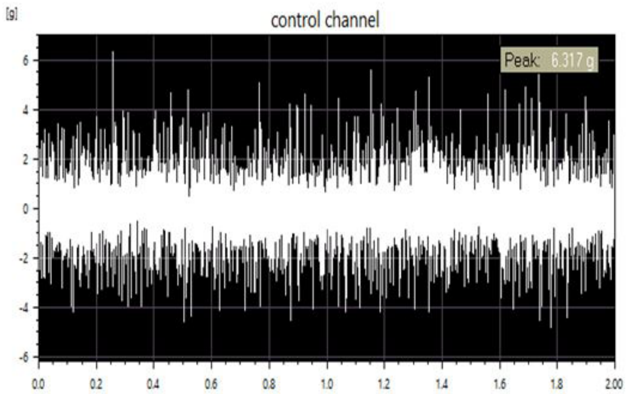

Figure.27b Random signal in time domain for Kurtosis 5

There was a higher occurrence of higher amplitude peaks when the Kurtosis was set to 5, with the highest positive peak of 8.2482 g and the negative peak of 9.6637 g. The crest factor for the highest positive peak was 8.4251 and the corresponding negative crest factor was 8.495.

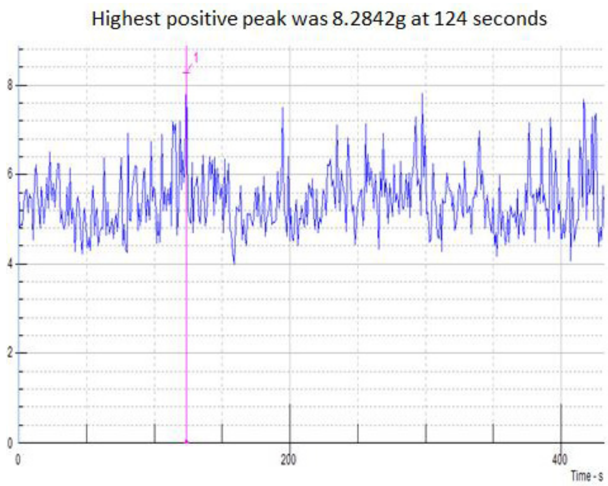

Figure.28 Positive peak time history for Kurtosis 5

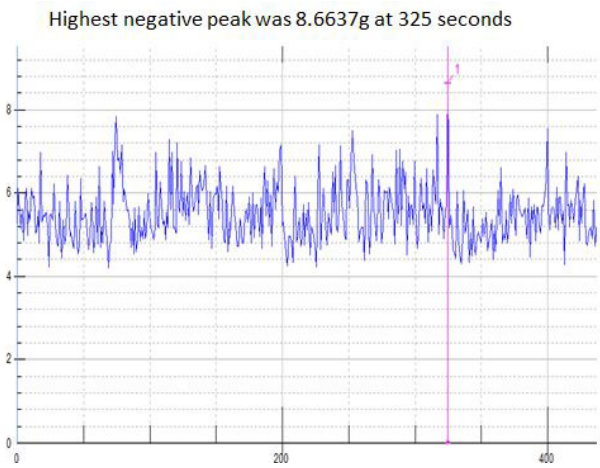

Figure.29 Negative peak time history for Kurtosis 5

Positive crest factor of 8.4251 at 124 seconds

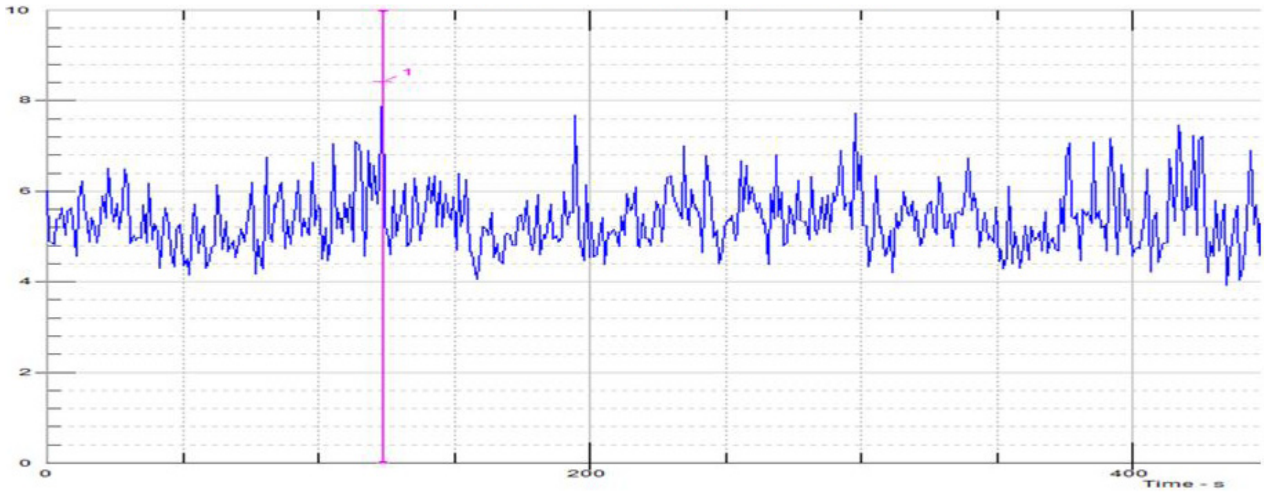

Figure.30 Positive crest factor time history for Kurtosis 5

Negative crest factor of 8.495 at 325 seconds

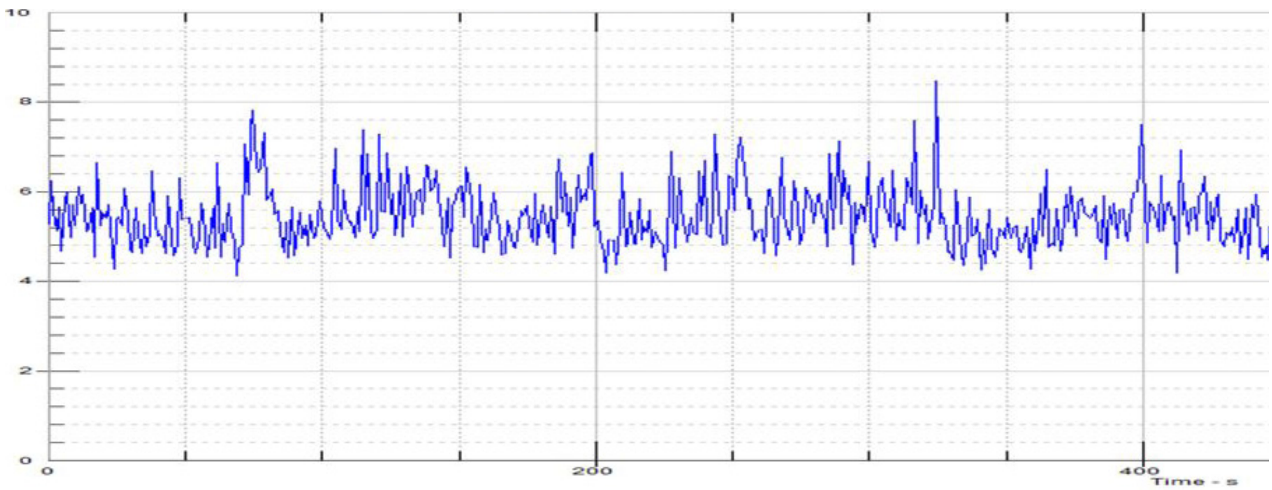

Figure.31 Negative crest factor time history for Kurtosis 5

Kurtosis 6

Test 1

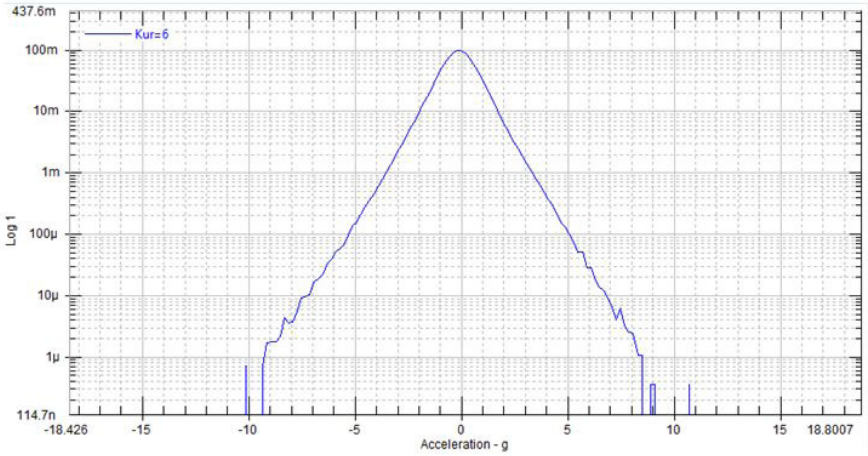

Figure.32a Probality Density Function for Kurtosis 6

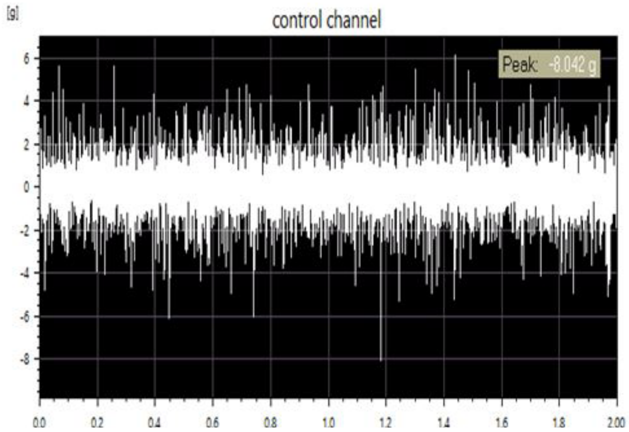

Figure.32b Random signal in time domain for Kurtosis 6

With the Kurtosis set to 6, there is a greater coverage of high amplitude values in the Probability Density Function, as evident in the results. There is significant increase in the occurrence of higher peak events. The highest positive peak reached a value of 11.295 g, while the negative peak was 9.6926 g. Respectively the calculated crest factors for the maximum amplitude events were 10.544 and 9.4985.

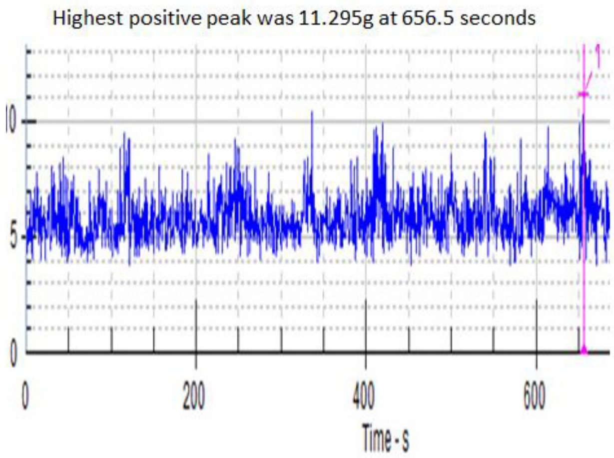

Figure.33 Positive peak time history for Kurtosis 6

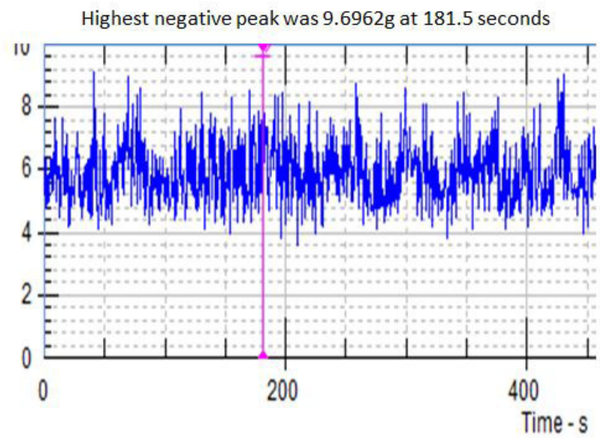

Figure.34 Negative peak time history for Kurtosis 6

Positive crest factor of 10.544 at 656.5 seconds

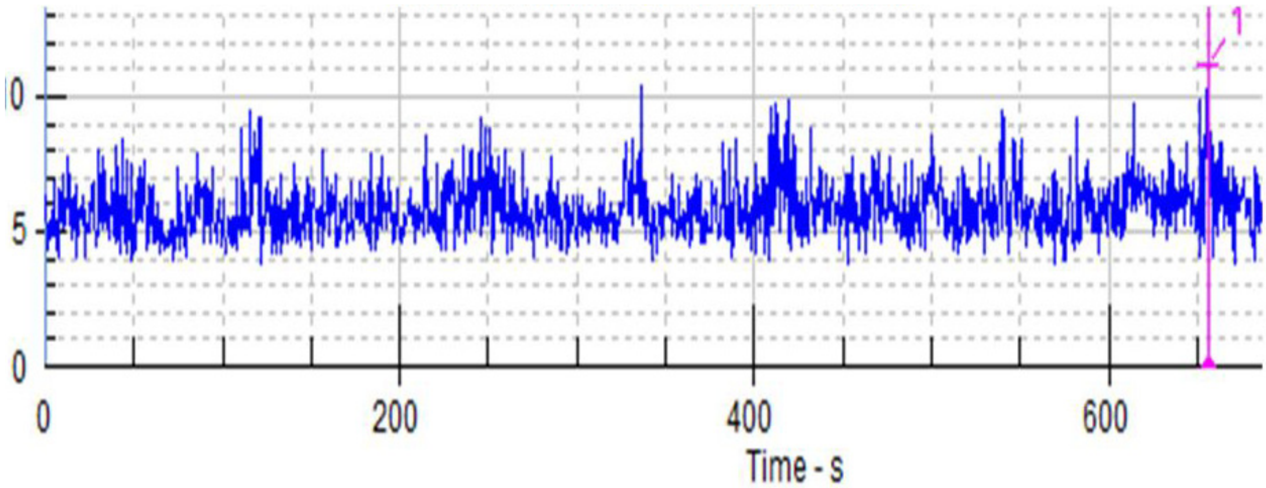

Figure.35 Positive crest factor time history for Kurtosis 6

Negative crest factor of 9.4935 at 181.5 seconds

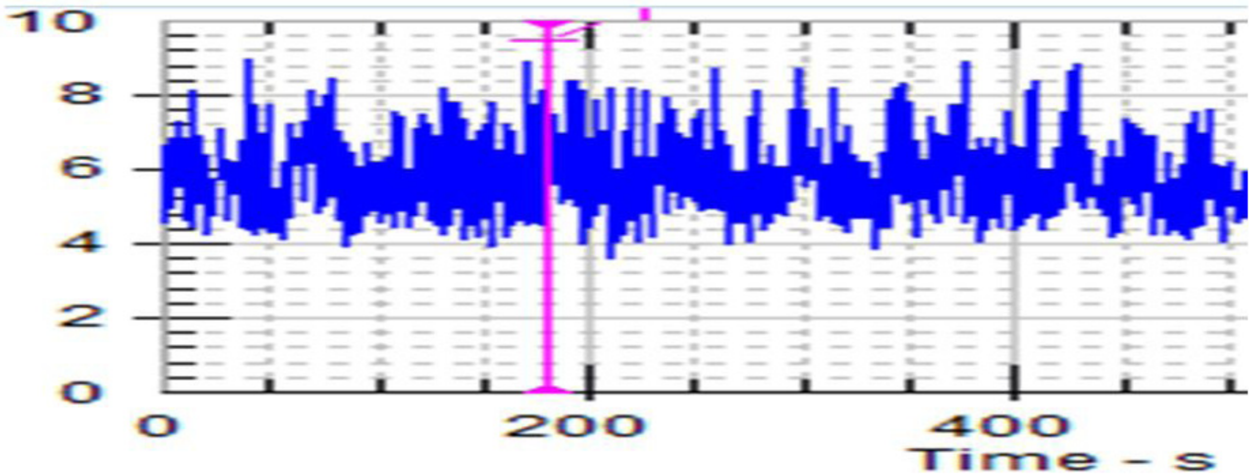

Figure.36 Negative crest factor time history for Kurtosis 6

Kurtosis 7

Test 1

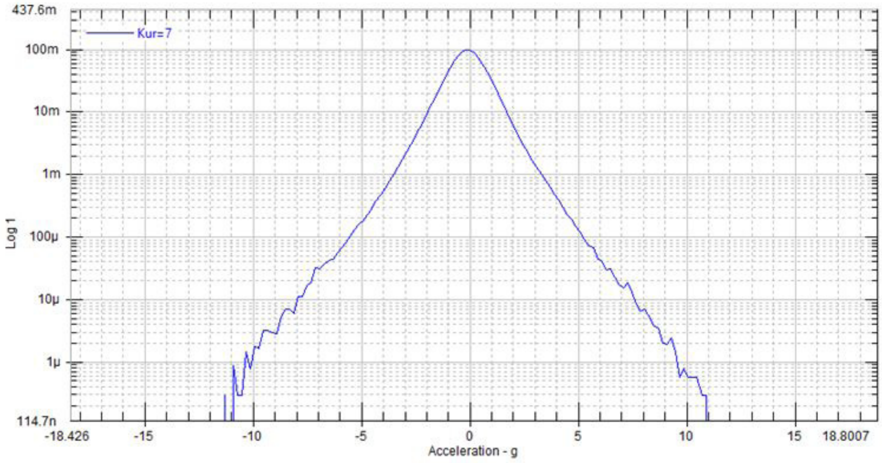

Figure.37a Probality Density Function for Kurtosis 7

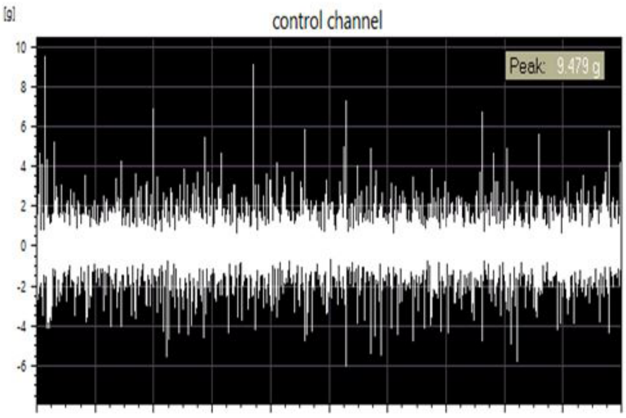

Figure.37b Random signal in time domain for Kurtosis 7

When the Kurtosis is set to 7, the highest positive peak generated in the random signal is 11.699 g, while the negative peak is 11.66376 g. Respectively the calculated crest factors for the maximum amplitude events were 11.125 and 11.195. From the results it is evident that there are strong similarities between the random signals with Kurtosis set to 6 and 7. This illustrates the point that even though Kurtosis increases the probability of higher peak events occurring, the random signal generated is still unique and non-deterministic, thus when and where peak events occur within the signal is not user controlled, due to the fact the signal is generated by a statistical means.

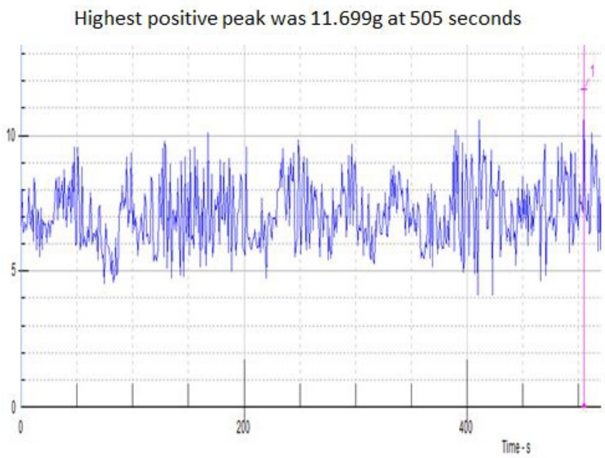

Figure.38 Positive peak time history for Kurtosis 7

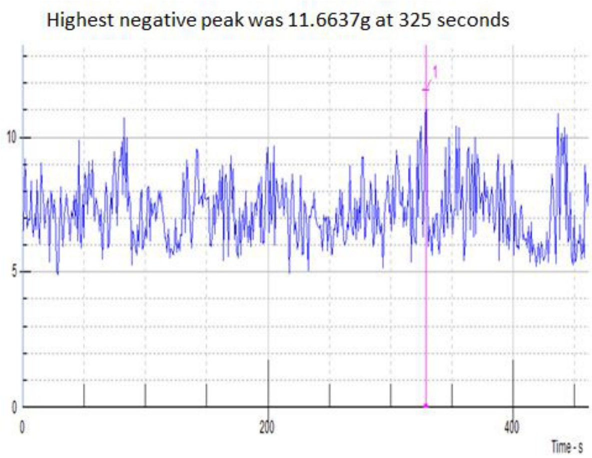

Figure.39 Negative peak time history for Kurtosis 7

Positive crest factor of 11.125 at 505 seconds

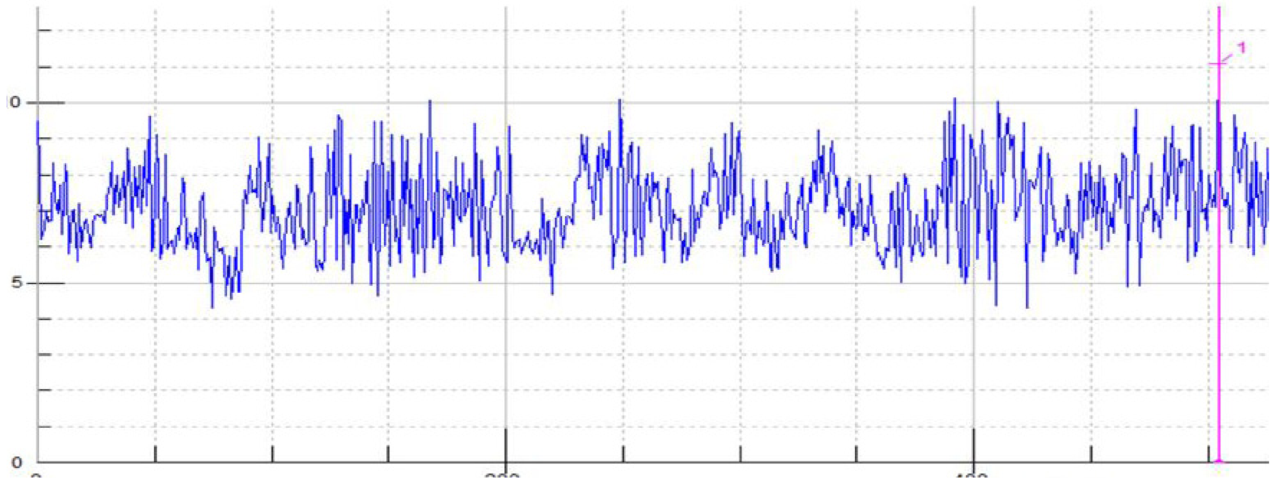

Figure.40 Positive crest factor time history for Kurtosis 7

Negative crest factor of 11.195 at 329 seconds

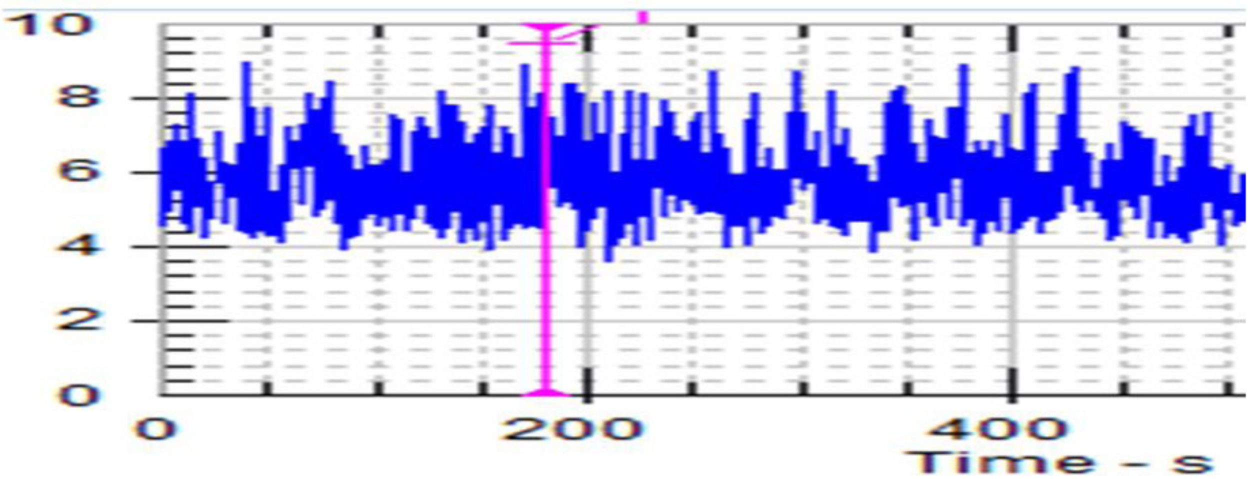

Figure.41 Negative crest factor time history for Kurtosis 7

Kurtosis 8

Test 1

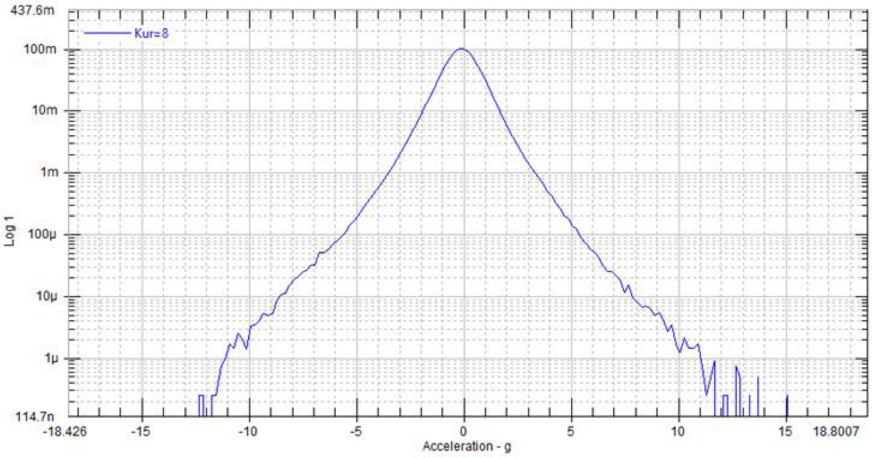

Figure.42a Probality Density Function for Kurtosis 8

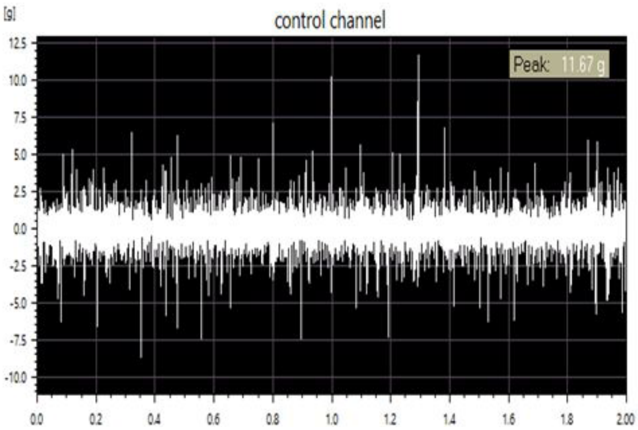

Figure.42b Random signal in time domain for Kurtosis 8

There was a higher occurrence of higher amplitude peaks when the Kurtosis was set to 8, with the highest positive peak of 13.680 g and the negative peak of 13.260 g. The crest factor for the highest positive peak was 12.357 and the corresponding negative crest factor was 12.652.

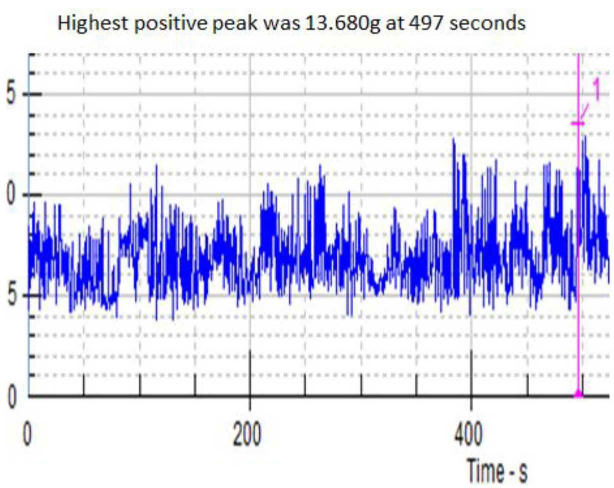

Figure.43 Positive peak time history for Kurtosis 8

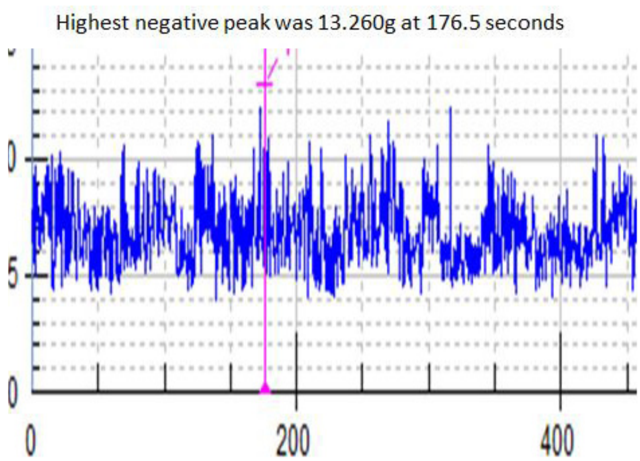

Figure.44 Negative peak time history for Kurtosis 8

Positive crest factor of 12.357 at 497 seconds

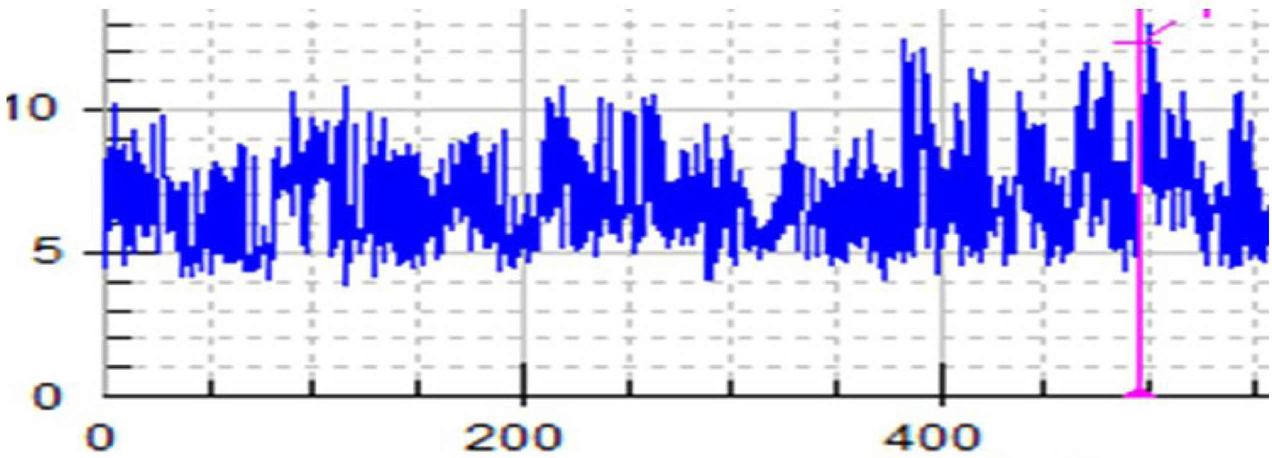

Figure.45 Positive crest factor time history for Kurtosis 8

Negative crest factor of 12.652 at 176.5 seconds

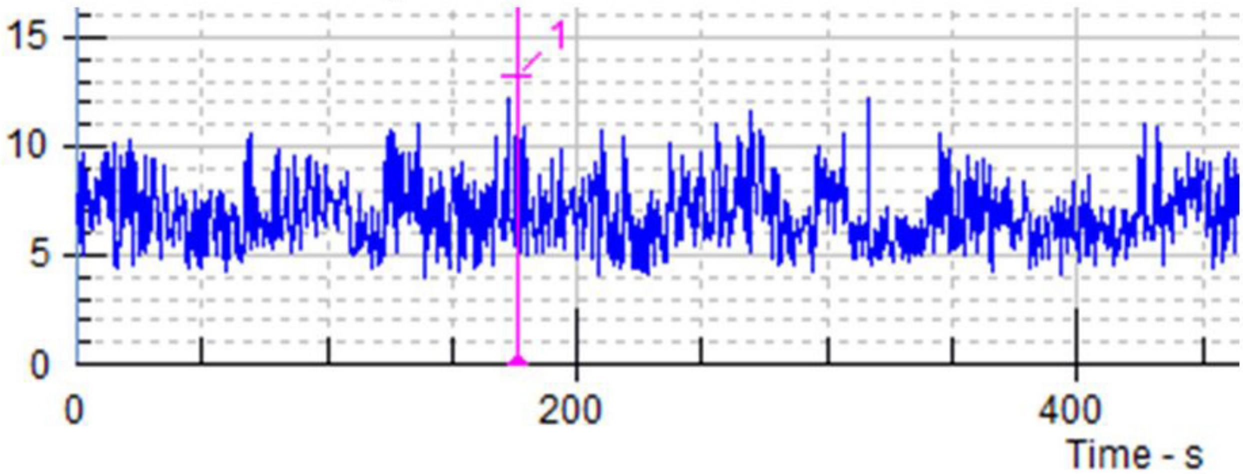

Figure.46 Negative crest factor time history for Kurtosis 8

Kurtosis 9

Test 1

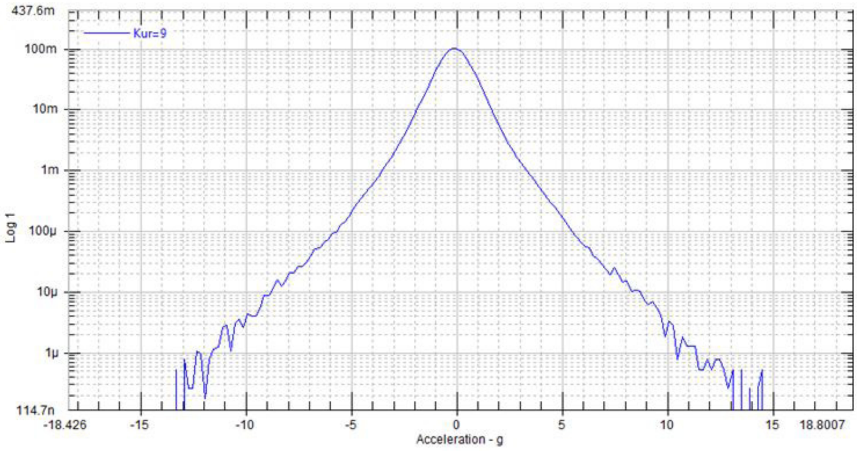

Figure.47a Probality Density Function for Kurtosis 9

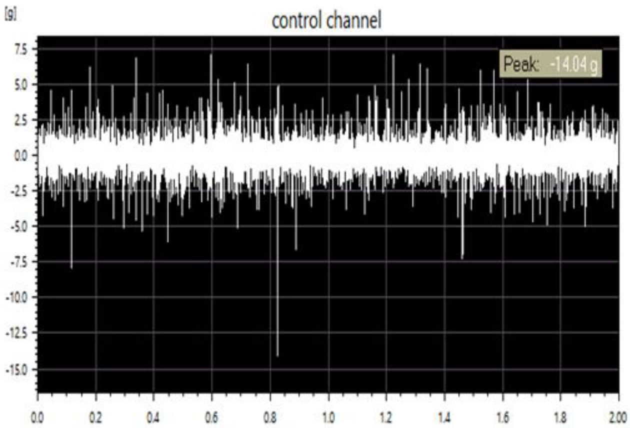

Figure.47b Random signal in time domain for Kurtosis 9

With the Kurtosis set to 9, there is a greater coverage of high amplitude values in the Probability Density Function, as evident in the results. There is significant increase in the occurrence of higher peak events. The highest positive peak reached a value of 14.891 g, while the negative peak was 13.273 g. Respectively the calculated crest factors for the maximum amplitude events were 14.566 and 12.363.

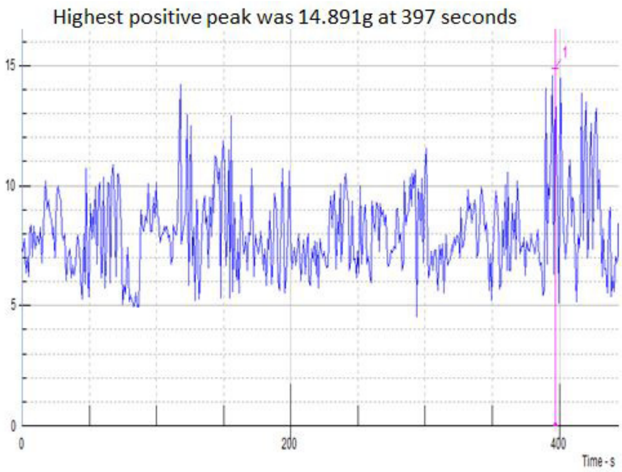

Figure.48 Positive peak time history for Kurtosis 9

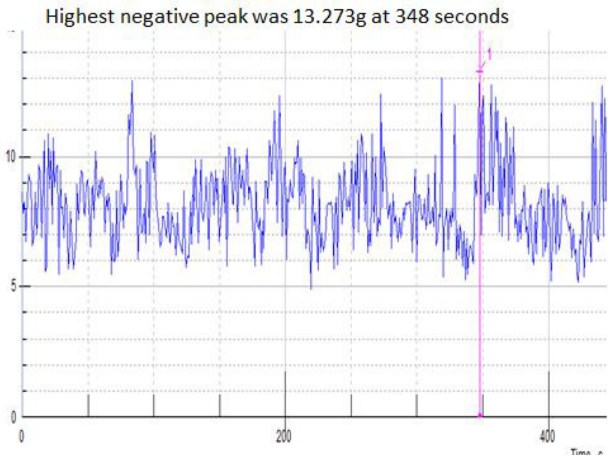

Figure.49 Negative peak time history for Kurtosis 9

Positive crest factor of 14.566 at 397 seconds

Figure.50 Positive crest factor time history for Kurtosis 9

Negative crest factor of 12.363 at 348 seconds

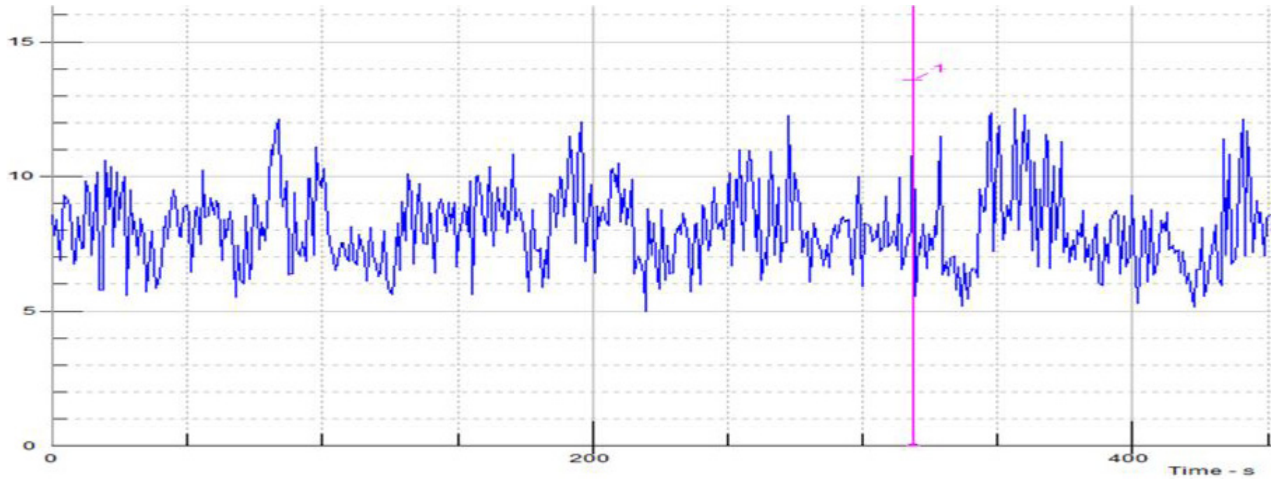

Figure.51 Negative crest factor time history for Kurtosis 9

Kurtosis 10

Test 1

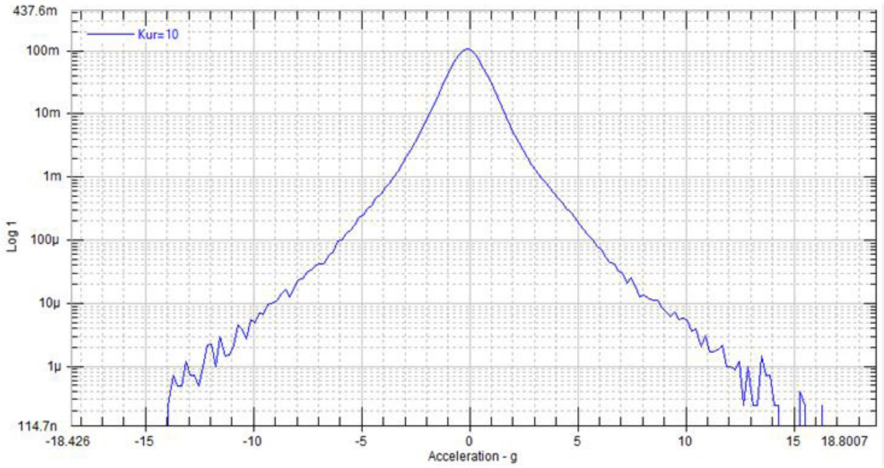

Figure.52a Probality Density Function for Kurtosis 10

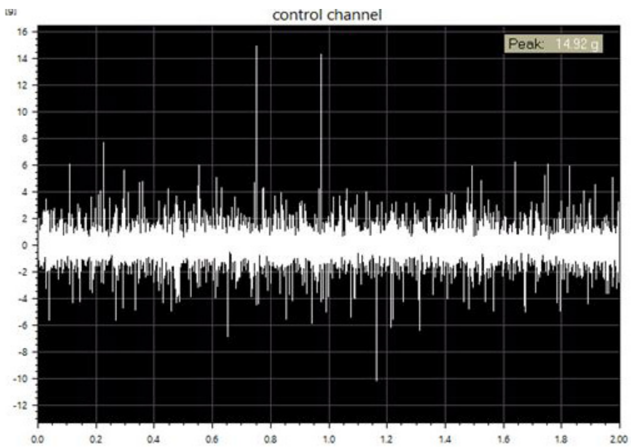

Figure.52b Random signal in time domain for Kurtosis 10

With the Kurtosis set to 10, the highest positive peak of the random signal reached a value of 15.697 g, while the negative peak was 13.954 g. Respectively the calculated crest factors for the maximum amplitude events were 14.364 and 13.3257.

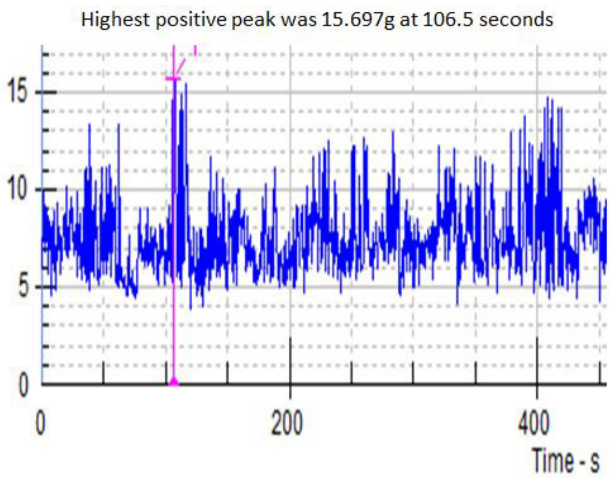

Figure.53 Positive peak time history for Kurtosis 10

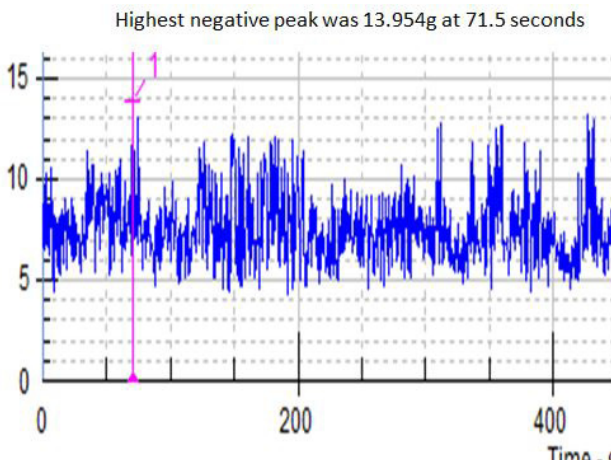

Figure.54 Negative peak time history for Kurtosis 10

Positive crest factor of 14.364 at 106.5 seconds

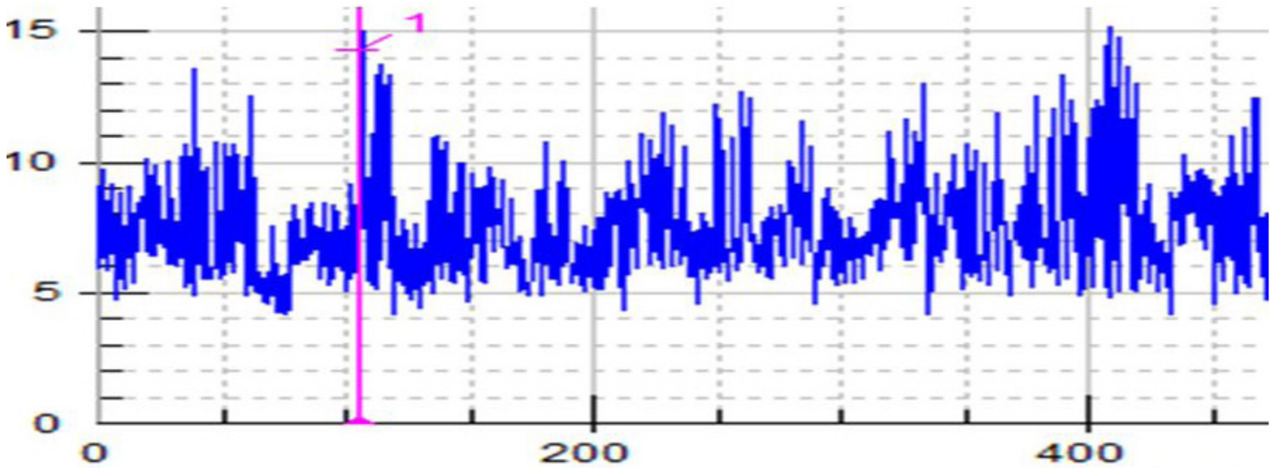

Figure.55 Positive crest factor time history for Kurtosis 10

Negative crest factor of 13.257 at 71.5 seconds

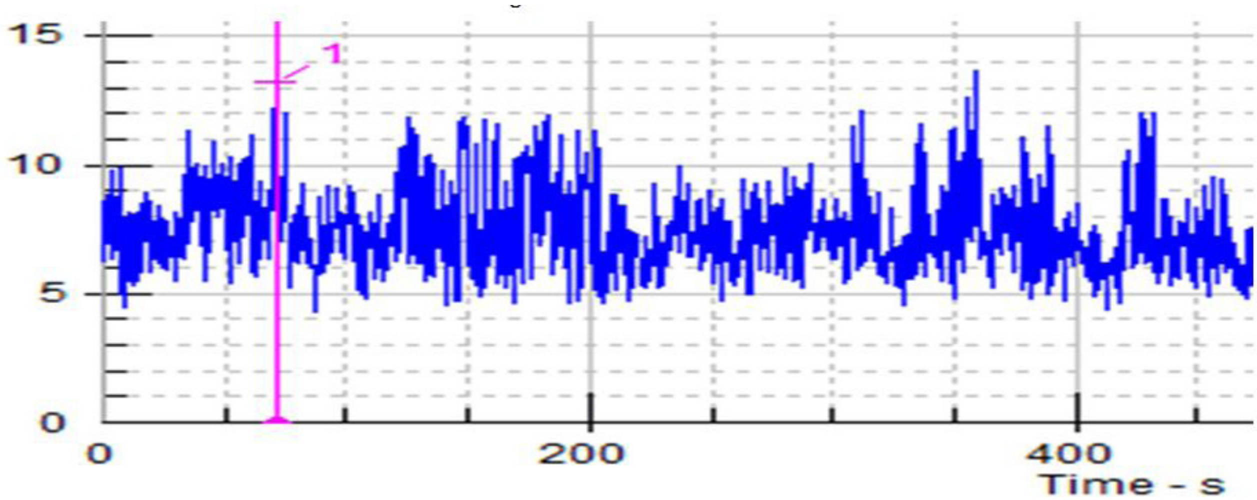

Figure.56 Negative crest factor time history for Kurtosis 10

Kurtosis 11

Test 1

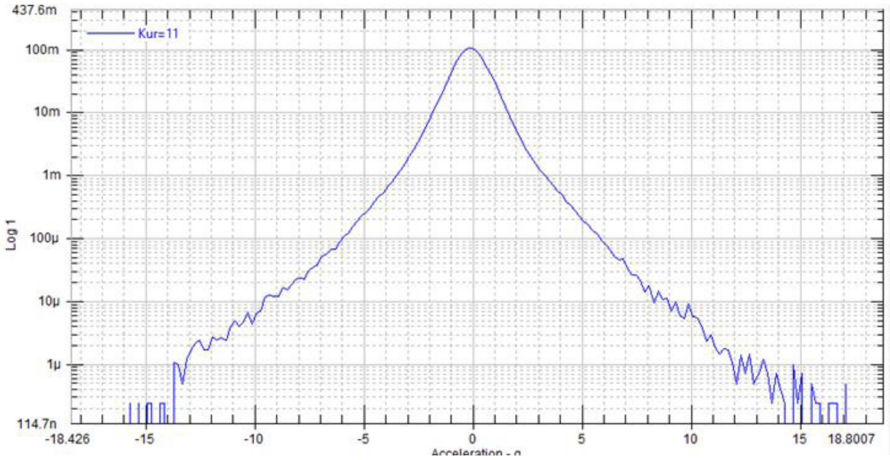

Figure.57a Probality Density Function for Kurtosis 11

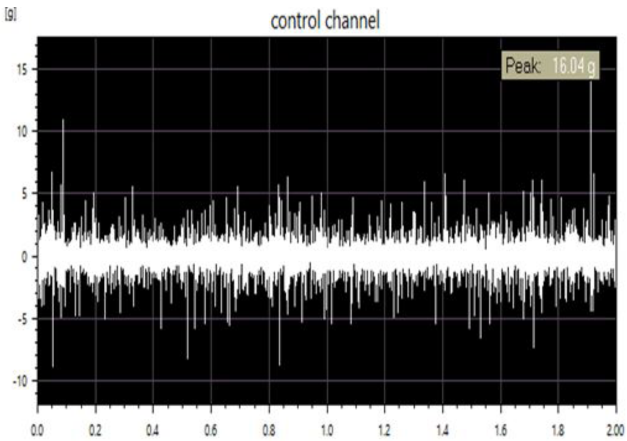

Figure.57b Random signal in time domain for Kurtosis 11

With the Kurtosis set to 11, the highest positive peak of the random signal reached a value of 17.206 g, while the negative peak was 15.958 g. Respectively the calculated crest factors for the maximum amplitude events were 17.242 and 16.087.

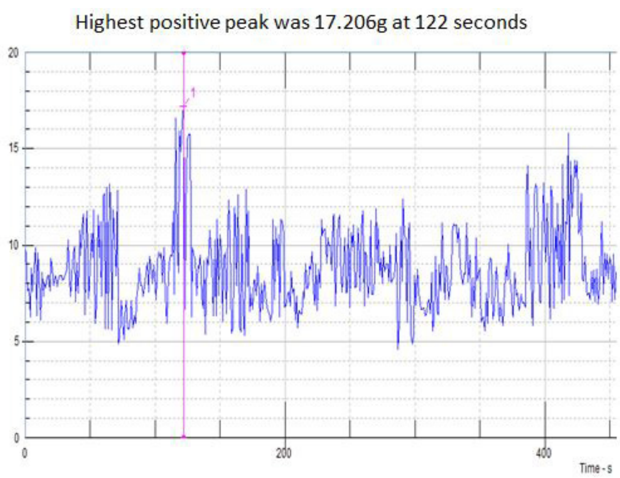

Figure.58 Positive peak time history for Kurtosis 11

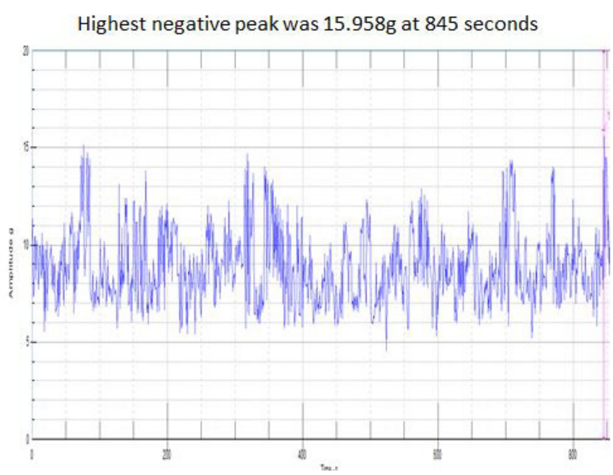

Figure.59 Negative peak time history for Kurtosis 11

Positive crest factor of 17.242 at 122 seconds

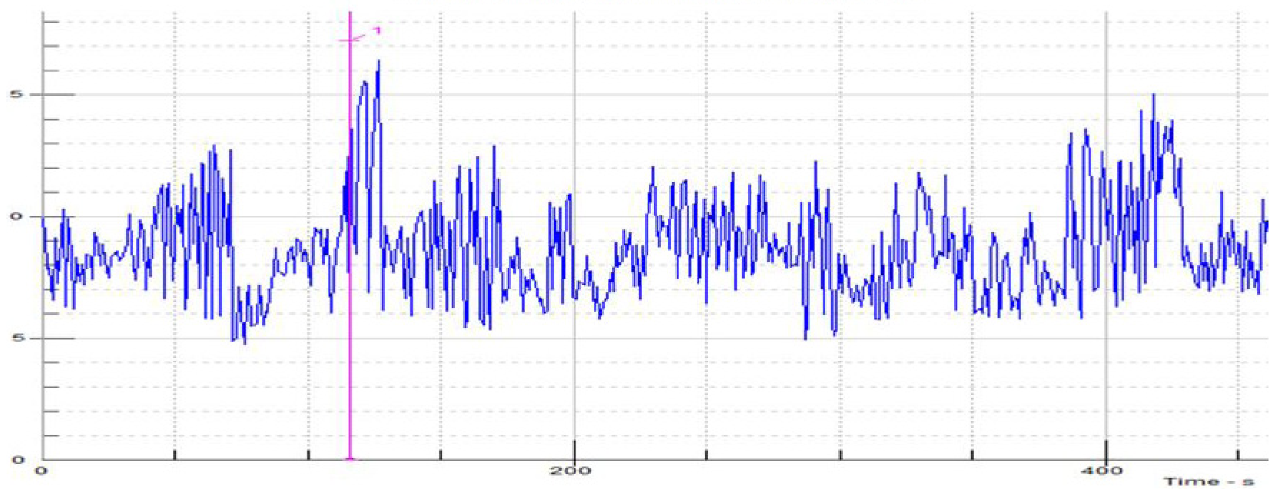

Figure.60 Positive crest factor time history for Kurtosis 11

Negative crest factor of 16.087 at 845 seconds

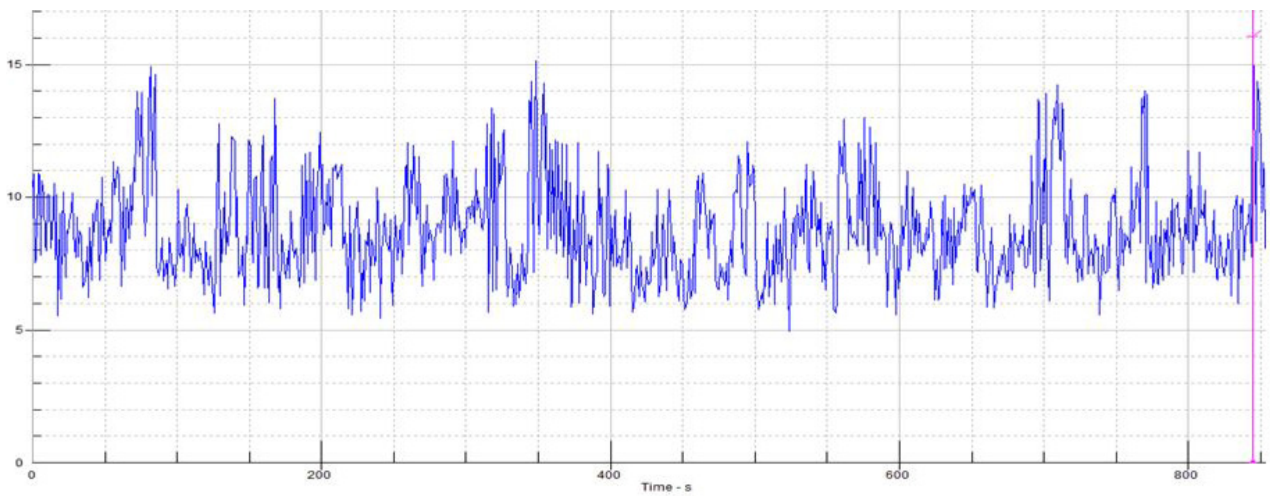

Figure.61 Negative crest factor time history for Kurtosis 11

Kurtosis 12

Test 1

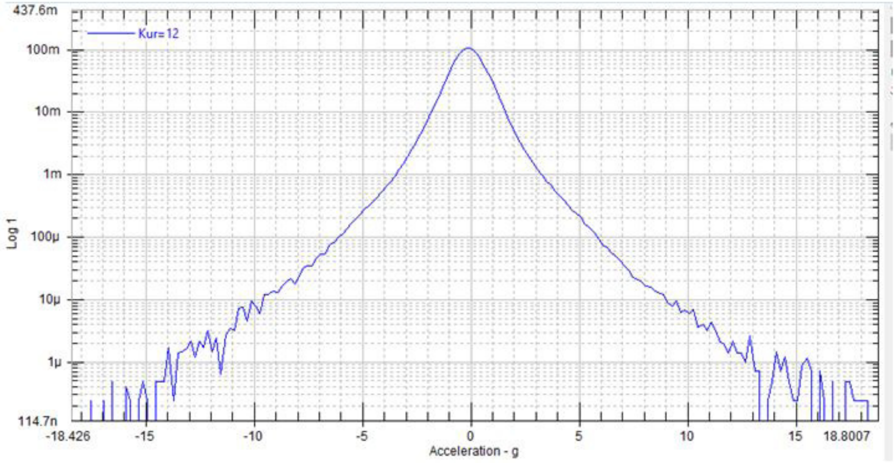

Figure.62a Probality Density Function for Kurtosis 12

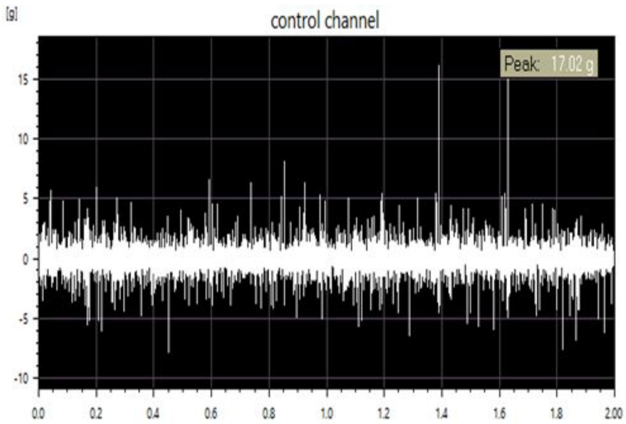

Figure.62b Random signal in time domain for Kurtosis 12

With the Kurtosis set to 12, the highest positive peak of the random signal reached a value of 17.843 g, while the negative peak was 18.5 g. Respectively the calculated crest factors for the maximum amplitude events were 16.470 and 16.969. It is also seen that even though higher peak levels are generated for increasing values of Kurtosis the probability of these occurring diminishes, meaning that higher Kurtosis is resulting from more regular events at somewhat lower levels. In practice this is advantageous to avoid excessive levels being needed in the shaker/amplifier.

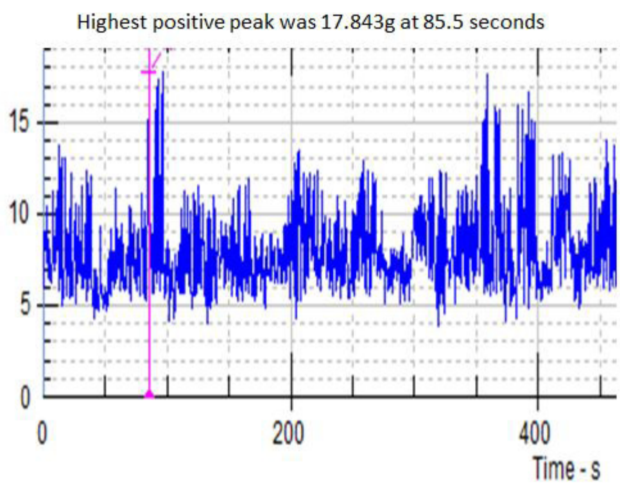

Figure.63 Positive peak time history for Kurtosis 12

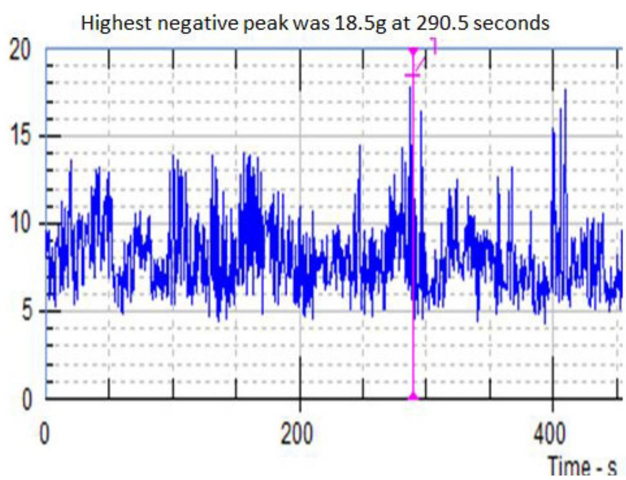

Figure.64 Negative peak time history for Kurtosis 12

Positive crest factor of 16.470 at 85.5 seconds

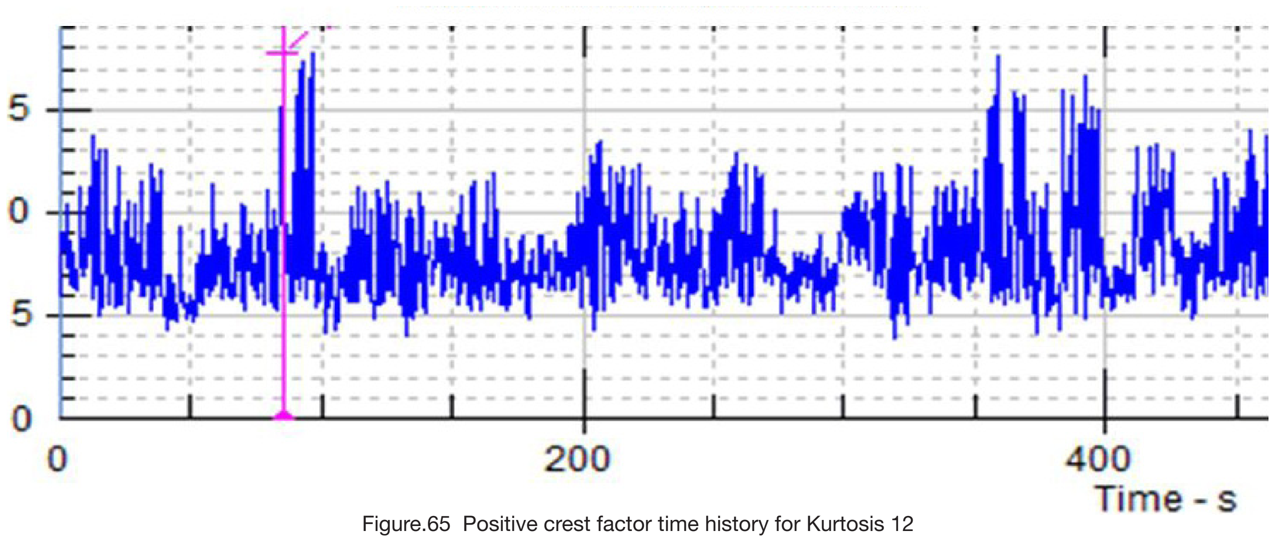

Figure.65 Positive crest factor time history for Kurtosis 12

Negative crest factor of 16.969 at 290.5 seconds

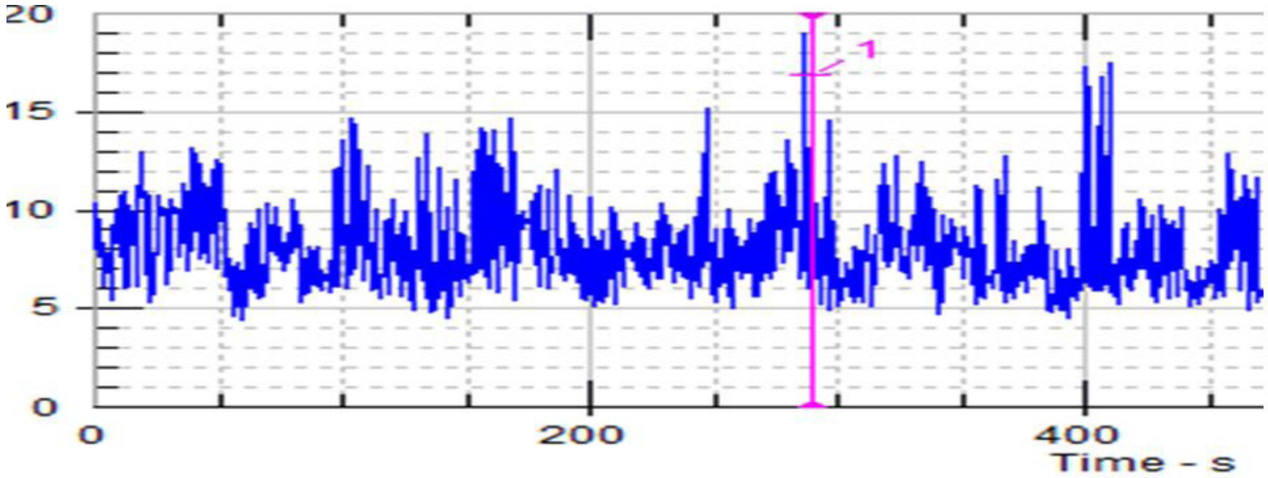

Figure.66 Negative crest factor time history for Kurtosis 12

Collected Kurtosis Results

	Test 1					Test 2					Test 3					Averages	
Kurtosis	Max positive peak(g)	Max negative peak(g)	positive cf	negative cf		Max positive peak(g)	Max negative peak(g)	positive cf	negative cf		Max positive peak(g)	Max negative peak(g)	positive cf	negative cf		Mean positive cf	Mean negative cf
3	5.0007	4.853	4.9059	4.8104		5.0612	4.8288	5.0164	4.8591		5.4434	5.2301	5.2133	5.1555		5.0452	4.94166667
4	7.5011	6.8004	7.3211	6.6536		6.6188	6.5201	6.6031	6.1467		6.6729	6.6634	6.5652	6.7239		6.8298	6.50806667
5	8.2842	8.6216	8.4251	8.495		9.9122	8.256	9.982	7.5668		9.332	8.7179	9.2986	7.9157		9.2352333	7.9925
6	11.295	9.6862	10.544	9.4935		11.814	11.091	12.146	11.503		10.53	10.543	10.841	10.606		11.177	10.5341667
7	11.699	11.6637	11.125	11.195		11.291	10.419	11.032	10.516		11.737	11.795	11.769	10.816		11.308667	10.8423333
8	13.68	13.26	12.357	12.652		13.066	11.96	12.704	12.756		13.868	12.104	12.509	12.668		12.523333	12.692
9	14.891	13.037	14.566	12.363		14.209	15.611	12.569	13.997		13.766	14.273	12.738	12.962		13.291	13.1073333
10	15.967	13.954	14.364	13.257		16.028	15.139	14.988	15.139		15.346	15.181	14.433	13.677		14.595	14.0243333
11	17.206	15.958	17.242	16.087		16.257	15.439	15.474	15.229		16.716	16.353	17.019	15.69		16.578333	15.6686667
12	17.843	18.5	16.47	16.969		18.993	18.312	16.482	16.202		18.12	16.636	17.841	16.918		16.931	16.6963333

Figure.67a Table showing the collected Kurtosis experiment results

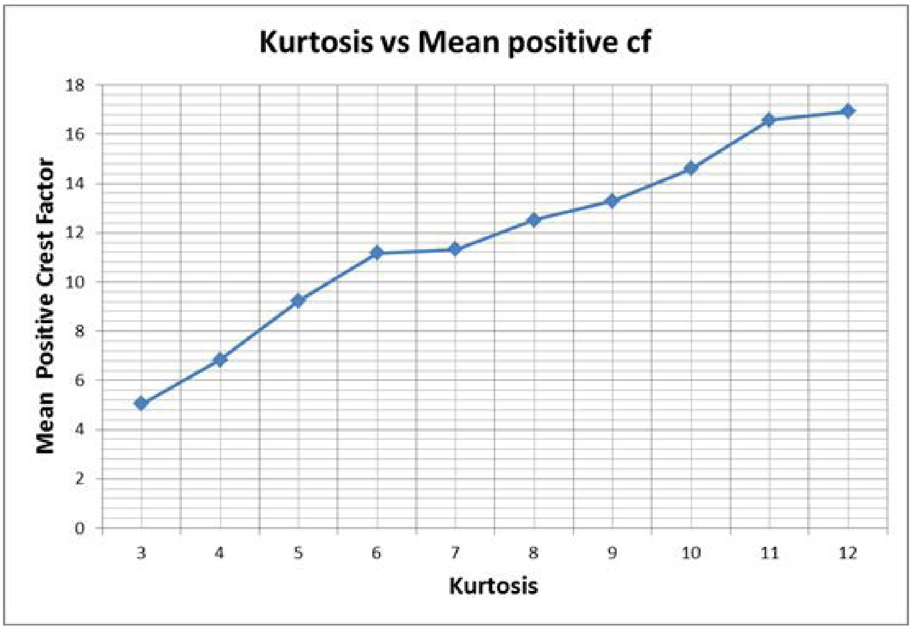

Figure.67b A graph of Kurtosis vs. mean positive crest factor

Test procedures were repeated three times for each Kurtosis level for a defined 1 g rms signal. Above is a table displaying the collected results, which exhibit the maximum positive and negative peak values and their corresponding crest factor values. The mean ratio of the crest factors are taken for each Kurtosis level.

As shown in the table at a Kurtosis level of 3, the set sigma clipping of 5 is maintained, as for an approx. 1 g rms signal the peak is approximately 5 g, thus the crest factor maintains the ratio of 5 ((Positive peak time history)/(rms time history)). Note: Fluctuations in the peak levels and cf are due to fluctuations in the rms signal, which is to be expected.

An interesting observation to make from the results on the table is that with each Kurtosis level, there was an increase in the mean positive and negative cf's. The maximum positive and negative peaks increased as the Kurtosis level increased as well. The only outliers to this observation occurred at Kurtosis levels 6 and 7 for test 1 and 2. This illustrates the point that even though Kurtosis increases the probability of higher peak events occurring, the random signal generated is still unique and non-deterministic, thus when and where peak events occur within the signal is not user controlled.

Conclusion

Simulating random vibration provides a better insight into the effects that real world motion has on objects and systems, especially compared to sine testing. However not all the properties of standard Gaussian random vibrations mimic or capture important aspects of the real world, such as high peak events, as discussed during the introduction. The new m+p Kurtosis control feature gives the user control over the high level content of the random vibration signal and this report describes the nature of these new signals in detail.

From the Kurtosis data collected a trend was observed that an increase of the Kurtosis values resulted in an increase in the crest factor ratio of the generated random vibrations, as well as an increase in the peak events being observed. This shows that the m+p Kurtosis feature increases the occurrence of higher peak events in a controlled way, thus providing m+p VibControl the ability to simulate a wider range of real world vibration environments.

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