MODAL SPACE - IN OUR OWN LITTLE WORLD

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Should more residuals be used to improve a curvefit? The results look better when many extra terms are used. This is an area that needs to be discussed..

When using frequency domain curvefitting techniques, many software packages allow the incorporation of extra terms in the polynomial in order to account for out of band effects. This is very useful in order to obtain accurate modal parameters. However, the user can specify many additional extra terms in order to improve the fit of the data. While this may "look" better, it is questionable where or not the parameters are actually better. So let's discuss the basic underlying equation and concept behind using residuals for modal parameter estimation. The basic frequency response equation can be written as

$$[H(s)]=\sum_{k=1}^{m}\frac{[A_k]}{(s-s_k)}+\frac{[A_k^*]}{(s-s_k^*)}$$

Now if we only write this equation over a band somewhere in the middle of the frequency response function, then there will be three different terms – one for the terms below the band of interest, the band of interest and one for the terms above the bands of interest. This is written as

$$[H(s)]=\sum_{\text{terms}}^{\text{lower}}\frac{[A_k]}{(s-s_k)}+\frac{[A_k^*]}{(s-s_k^*)}$$

$$\sum_{k=i}^{j}\frac{[A_k]}{(s-s_k)}+\frac{[A_k^*]}{(s-s_k^*)}$$

$$\sum_{\text{terms}}^{\text{upper}}\frac{[A_k]}{(s-s_k)}+\frac{[A_k^*]}{(s-s_k^*)}$$

And we often write this equation with only the modes of interest, over the band of interest, and apply extra terms called residuals to compensate for out of band effects and is written as

$$[H(s)]=LR+\sum_{k=i}^{j}\frac{[A_k]}{(s-s_k)}+\frac{[A_k^*]}{(s-s_k^*)}+UR$$

A typical frequency response function illustrating this is shown in Figure 1.

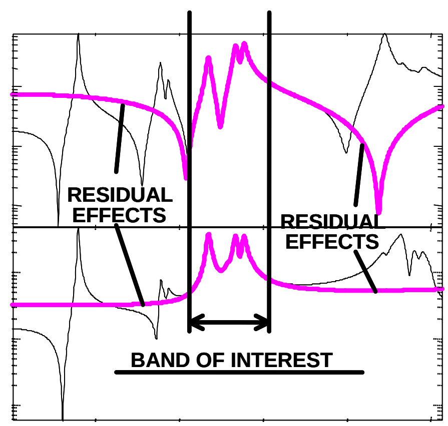

Figure 1 – Illustration of FRF with Band of Interest

In order to describe the residual terms, it is advantageous to look at the single degree of freedom displacement frequency response function. Figure 2 shows that frequencies below that of the resonant frequency are basically described by a dominant stiffness term and that the frequencies above that of the resonant frequency are basically described by a dominant mass term. It is this basic fact that allows the frequency response function to be written with the band of interest along with a lower residual term (LR) and an upper residual term (UR). Usually 4 extra residual terms in a polynomial curvefitter are sufficient in order to approximate these terms.

So now let's use a measurement to illustrate what happens when residual terms are overspecified to extract parameters. A simple 6 DOF model with a band of four modes bounded by two dominant modes will be used.

Now a curvefit for the four modes in the middle of the band is performed using the typical residual terms in most polynomial curvefitters (4 extra terms) and the fit is seen in Figure 3. Notice that the fit is reasonable but it doesn't fit the data well over all over all frequencies – at least from a visual perspective. Because the fit only used 4 extra residual terms, the next curvefit performed uses 10 extra residual terms and is seen in Figure 4. Now this fit appears better overall – from a visual standpoint anyway. And just to illustrate a point, the fit is also done with a simple SDOF shown in Figure 5.

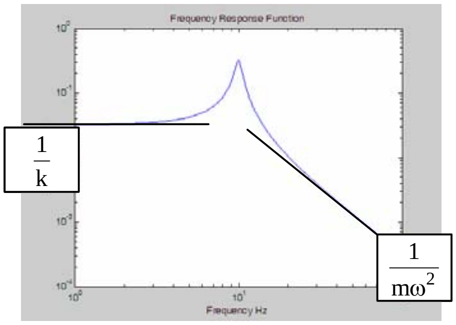

Figure 2 – Single DOF System with Residual Terms

But in order to really evaluate these fits, the extracted data needs to be compared to the actual parameters that were used to develop the frequency response functions. Table 1 lists the frequencies, damping and residues for the four modes along with the parameters extracted from both curvefit approaches.

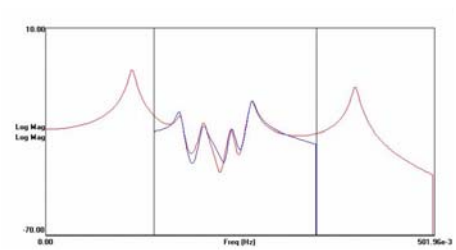
Figure 3 – Curvefit with 4 Residual Terms

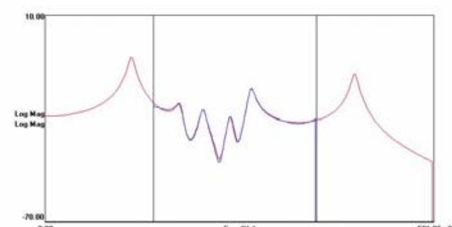
Figure 4 – Curvefit with 10 Residual Terms

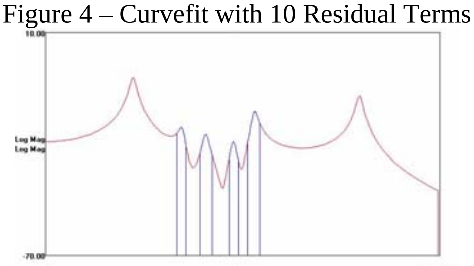
Figure 5 – Curvefit with SDOF Polynomial

Once the data in Table 1 is evaluated and assessed, it becomes clear that the addition of extra residual terms does not improve the parameter estimation overall and actually might degrade the results somewhat. Also note that the SDOF produces the best results overall. This brings up the point that the modal parameter estimation process is about extracting reasonable parameters to describe the system characteristics – and not necessarily about making curves that overlay on top of each other. In all the years of estimating modal parameters, it has become very clear that the overspecification of residual terms is only trying to compensate for noise or imperfections in the frequency response functions obtained. The overspecification of residual terms is not considered to be the reasonable approach for extracting modal parameters. The default residual terms specified in most commercially available software packages are reasonable for most curvefitting applcations. If many extra residual terms are needed to fit measured frequency response functions to "look better", then it is likely that the measured functions are contaminated with noise or imperfections and better measurements are likely needed.

Table 1 –Frequencies/Damping/Residues

Exact Analytical Results

Mode	Frequency Hz	Damping % Critical	Residue Value
1	0.173	2.46	311
2	0.203	1.95	233
3	0.239	1.55	159
4	0.265	1.49	595

4 Modes Extracted With 4 Residual terms

Mode	Frequency Hz	Damping % Critical	Residue Value
1	0.173	2.17	349
2	0.202	2.22	223
3	0.239	1.65	149
4	0.265	1.51	596

4 Modes Extracted With 10 Residual terms

Mode	Frequency Hz	Damping % Critical	Residue Value
1	0.173	2.66	348
2	0.203	1.91	231
3	0.238	1.43	137
4	0.265	1.50	584

4 Modes Extracted With SDOF Approach

Mode	Frequency Hz	Damping % Critical	Residue Value
1	0.173	2.30	314
2	0.203	1.96	234
3	0.239	1.68	159
4	0.265	1.50	594

I hope that these simple cases illustrate some important points regarding modal parameter estimation. Overspecifying residual terms is not the preferred approach for extracting accurate parameters. If you have any more questions on modal analysis, just ask me.