Illustration by Mike Avitabile

I hear mode participation all the time. What does that really mean? OK – let's discuss this.

So people always talk about mode participation but maybe it really isn't clear what they are referring to when they talk about it. So let's discuss what this concept is all about and put it in terms that might make more sense.

But to put it in context, let's write the equation of motion

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\}$$

and recognize that the modal transformation is obtained from the eigen solution with the physical coordinate {x} is related to the modal coordinate {p} using the collection of modal vectors [U]

$$\{x\}=[U]\{p\}=\{u_1\}p_1+\{u_2\}p_2+\{u_3\}p_3+\cdots$$
$$\text{with } [U]=[\{u_1\}\quad\{u_2\}\quad\{u_3\}\quad\cdots]$$

and then further remember that the modal space equation is

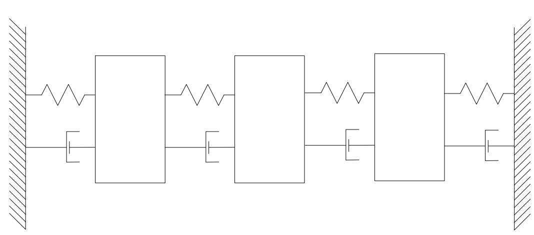
$$\begin{bmatrix}\bar{m}_1 & & \\ & \bar{m}_2 & \\ & & \ddots\end{bmatrix}\begin{Bmatrix}\ddot{p}_1\\ \ddot{p}_2\\ \vdots\end{Bmatrix}+\begin{bmatrix}\bar{c}_1 & & \\ & \bar{c}_2 & \\ & & \ddots\end{bmatrix}\begin{Bmatrix}\dot{p}_1\\ \dot{p}_2\\ \vdots\end{Bmatrix}+\begin{bmatrix}\bar{k}_1 & & \\ & \bar{k}_2 & \\ & & \ddots\end{bmatrix}\begin{Bmatrix}p_1\\ p_2\\ \vdots\end{Bmatrix}=\begin{Bmatrix}\{u_1\}^T\{F\}\\ \{u_2\}^T\{F\}\\ \vdots\end{Bmatrix}$$

So the most important thing to understand is that the right hand side of this equation has the transpose of the mode shape times the physical force vector that is applied on the structure. So if you looked at a particular mode of interest, then you would see that the mode shape values have a strong effect on how much of that physical force is allocated or approporiated to the particular mode of interest.

What I mean by that is that if the value of the mode shape is large associated with the particular degree of freedom where the force is applied then that particular mode will get a lot of force appropriated to it in modal space. On the other hand if the value of the mode shape is small then there will be much less force appropriated to that particular mode in modal space. And if the value of the mode shape is zero then there will be no force allocated to that mode in modal space – this means that this particular mode has no contribution to the response because it sees no force applied on that particular modal oscillator in modal space.

So the modal transformation equation identifies how to uncouple all the coupled set of physical equations and it also identifies how much of the physical force is allocated for each of the modal oscillators in modal space. The larger the force that is applied to a particular modal oscillator, the large the response (in general) and the more that particular mode contributes or participates in the total response of the system.

So let's try a simple 3 dof system and see what force gets appropriated to modal space. Here is the model, with the equation of motion in physical space



$$\begin{bmatrix}1 & & \\ & 1 & \\ & & 1\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\end{Bmatrix}+\begin{bmatrix}0.2 & -0.1 & \\ -0.1 & 0.2 & -0.1\\ & -0.1 & 0.2\end{bmatrix}\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{Bmatrix}+\begin{bmatrix}20000 & -10000 & \\ -10000 & 20000 & -10000\\ & -10000 & 20000\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\\ x_3\end{Bmatrix}=\begin{Bmatrix}f_1\\ f_2\\ f_3\end{Bmatrix}$$

Fig 1 – Simple 3DOF Model and Equation of Motion

Now an eigensolution for this will result in frequencies and mode shapes as

$$\left[\Omega^2\right]=\begin{bmatrix}5858 & & \\ & 20000 & \\ & & 34142\end{bmatrix}$$

$$[U]=[\{u_1\} \quad \{u_2\} \quad \{u_3\}]=\left[\begin{Bmatrix}0.500\\0.707\\0.500\end{Bmatrix} \quad \begin{Bmatrix}0.707\\0\\-0.707\end{Bmatrix} \quad \begin{Bmatrix}-0.500\\0.707\\-0.500\end{Bmatrix}\right]$$

Now let's consider two different forcing functions.

$$F=\begin{Bmatrix}f_1\\0\\0\end{Bmatrix} \quad \text{and} \quad F=\begin{Bmatrix}0\\f_2\\0\end{Bmatrix}$$

For the first case with just f_1 applied, the force on mode 1 would be $0.5*f_1$, the force on mode 2 would be $0.707*f_1$ and the force on mode 3 would be $-0.5*f_1$. Now there are different allocations of the physical force on each of the modal oscillators which is controlled by the value of the mode shape associated with the degree of freedom where the force is applied.

Now for the second case with just f_2 applied, the forces on each of the modal oscillators would be $0.707*f_2$, $0.0*f_2$ and $0.707*f_2$ for each of the three modes. Notice that mode 2 sees no force because the value of the mode shape is zero for the degree of freedom associated with the force for mode 2. So we can say that mode 2 does not participate in the response of the system. Its modal participation is zero. But that doesn't mean that mode 2 doesn't exist – it just means that it doesn't contribute to the response of the system for this particular loading scenario (but it certainly has contribution for the first loading condition).

So let me try to give a little example to explain this a little better. Let's say that you are a cook in a restaurant. And imagine that you have a lot of different recipes that you might make. You also have a lot of ingredients and spices that could be used in all the different recipes that you make. Here is my question. Do you use all of your spices in all of your recipes with equal amounts of all the spices. No. You use varying amounts of spices in each recipe. And in some recipes there are many spices that are never even used. What I am trying to say is that you don't use all of your spices all the time. Only certain spices "participate" in each recipe and to varying amounts.

Now if I am cooking some French dish then there are certain spices that will be more predominant then if I was making a Japanese dish. But my spice cabinet still has all the spices I could possibly need for all the different types of dishes that I may cook. But that doesn't mean that I use every spice I have just because they are in the cabinet. And one particular spice isn't in every recipe (except if you are cooking Italian, then garlic goes in everything!) But I think you get the idea now.

BASIL AND GARLIC MINESTRONE
3 ounces pancetta, finely chopped, 3 cloves garlic, 1 cup olive oil, pinch of oregano, 1/2 cup Italian tomatoes, 1 tsp chicken bouillon, 1 tablespoon salt, pinch of pepper, 10 ounces penna pasta, 3 tablespoons fresh basil, 1 cup Parmesan cheese

Another good example would be that of an orchestra. There are many instruments that are available in the orchestra. Every possible score won't use all the possible instruments all the time. In fact as the particular score proceeds there will be varying contributions from each of the instruments. Sometimes the horns will be dominant and sometimes the strings will be the strongest instrument. And as the score progresses, each of the instruments will participate to varying degrees depending on the particular musical arrangement. And sometimes some of the scores will not need any contribution from certain instruments (like the guy in the back with the triangle). But the orchestra always consists of all the members of the orchestra – but all of the instruments have varying participation for each of the different scores that the orchestra plays.

Well the same is true of the response of any structural system. The total response of the system is made up from linear combinations of a subset of the total number of modes that possibly exist in the system. Not all modes contribute to the response for every forcing condition that might exist. Only certain modes may contribute substantially and some others may contribute a little bit and yet some others may not have any contribution whatsoever. This amount of contribution of all the modes changes depending on what loading condition is considered. So certain modes have different modal participations depending on what loading scenario is considered.

But the important thing to understand is that the mode shape plays an important part in determining the amount of force that is appropriated to the modal oscillator in modal space along with the distribution of force on the physical system. So understanding the modes shapes also helps to identify the force applied to all the modes of the system.

I hope this explanation helps you to understand modal participation a little better. If you have any other questions about modal analysis, just ask me.