MODAL SPACE - IN OUR OWN LITTLE WORLD

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If we perform a roving impact hammer test and impact many points, is there any possibility of missing a mode?
Well… you need to be careful where you place the accelerometer.

I am glad you asked this question because it is a very important consideration when perfoming a modal test. Let's discuss a few things in regards to your question.

Now a roving hammer with a stationary accelerometer is one way to run an impact test that is commonly used. The other way an impact test can be performed is by keeping the hammer stationary and moving the accelerometer. Both are acceptable ways to run this test and because of reciprocity, there really isn't a difference from a theoretical standpoint. In fact if you consider the measurements made you will fill one row of the FRF matrix when you have a roving hammer and you will fill one column of the FRF matrix if you have a roving response transducer. This is shown schematically in Figure 1 along with the reciprocal measurements.

Anytime you run a modal test, you always have to be careful to avoid having the reference located at the node of a mode. This is the most important consideration.

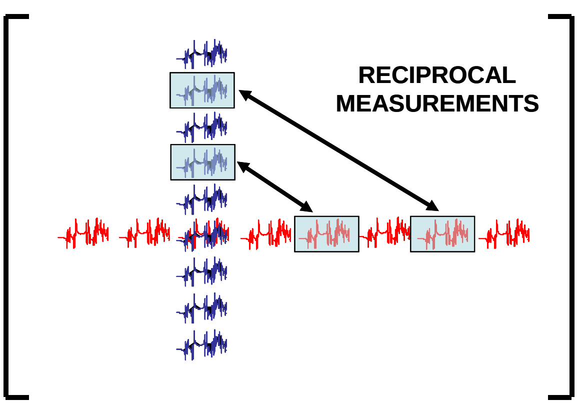


Figure 1 – FRF Matrix Showing Roving Impact (Red Row) and FRF Matrix Showing Roving Response (Blue Column)

In order to understand this, a few very basic equations describing the FRF equation need to be presented.
Now a single FRF "ij" measurement can be written in summation form in terms of residues as

$$h(s)_{ij}\Big|_{s=j\omega} = h_{ij}(j\omega) = \sum_{k=1}^{m} \frac{a_{ijk}}{(j\omega - p_k)} + \frac{a^*_{ijk}}{(j\omega - p^*_k)}$$

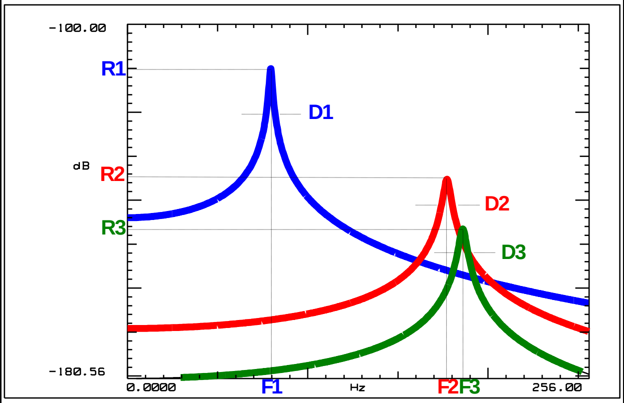
But in this residue form of the equation, it is not so easy to realize what the residues imply. But if I write this equation using the residues expressed as mode shapes then

$$h(s)_{ij}\Big|_{s=j\omega} = h_{ij}(j\omega) = \sum_{k=1}^{m} \frac{q_k u_{ik} u_{ij}}{(j\omega - p_k)} + \frac{q^*_k u^*_{ik} u^*_{jk}}{(j\omega - p^*_k)}$$

The resulting FRF for this equation might look like the FRF shown in Figure 2 where the two forms of the FRF formulation are shown (and colored in blue and red for clarity of the individual mode contribution for each term of the FRF summation); in Figure 2, the equations above have been expanded for the first two terms of the summation to show the contribution that each mode makes to the total FRF. The important thing to realize is that the FRF is made up of the summation of each of the individual modes.

When we write this equation in terms of the mode shapes, it becomes very clear how the mode shapes of the structure have a strong influence on the amplitude of the FRF for a particular "ij" term. The residue is basically developed from a scaling coefficient, q, and the ***value of the mode shape at the output response location*** times the ***value of the mode shape at the input excitation location***. With that said, I think it becomes very clear that if either the output location or input location mode shape value is zero (that is, located at the node of the mode), then there will be no amplitude for that particular mode.

$$h_{ij}(j\omega) = \frac{q_1 u_{i1} u_{j1}}{(j\omega - p_1)} + \frac{q_1^* u_{i1}^* u_{j1}^*}{(j\omega - p_1^*)} + \frac{q_2 u_{i2} u_{j2}}{(j\omega - p_2)} + \frac{q_2^* u_{i2}^* u_{j2}^*}{(j\omega - p_2^*)} + \cdots$$



$$h_{ij}(j\omega) = \frac{a_{ij1}}{(j\omega - p_1)} + \frac{a_{ij1}^*}{(j\omega - p_1^*)} + \frac{a_{ij2}}{(j\omega - p_2)} + \frac{a_{ij2}^*}{(j\omega - p_2^*)} + \cdots$$

Figure 2 – FRF Summation

So now that we realize this, it is also clear that any time we measure at the node of a mode then there will be no apparent amplitude in the FRF measurement related to that mode – and it doesn't matter if it is the excitation or response location.

And I should also point out that if the reference is located very close to the node of the mode then the amplitude of the FRF will be very low for that particular mode. Actually the 3rd mode shown in green in Figure 2 is a prime example of that. That amplitude is low compared to the 1st and 2nd mode because the value of the mode shape for the input and/or output location is much smaller than that for the 1st and 2nd mode; that's why the amplitude is much lower.

But as far as the original question you asked, what really matters is the reference location - whether it be the stationary hammer as a reference or the stationary accelerometer as a reference. If the reference is located at the node of a mode then there will be no apparent response in the FRF for that particular mode.

Now in order to conduct a good modal test, we need to have a pretty good idea of what the mode shapes are for all the modes of interest so that a proper reference can be selected.

But many times in more complicated structures or structures with very directional modal characteristics, it may be very hard if not impossible to select one location where it is easy to see all the modes from that one reference location. That is why so many times, we conduct modal tests with several references. This way we have the ability to see all the modes from a collection of different reference locations.

Often times, we will use 3 accelerometers when we have a 4 channel acquisition system (or 7 accelerometers if we have an 8 channel acquisition system), and perform a roving hammer impact modal test with stationary accelerometers at different locations on the structure. That way we have a much better chance to make sure that we can see all the modes from all the different reference locations.

Hopefully we can pick 3 (or 7 locations) from which all the references are not located on the nodes of modes for all the references. In fact you might think that it is almost impossible to pick that many references and have all of them be on the node of a mode all simultaneously.

Well as luck might have it, there was one modal test I saw where there were 9 references used with a roving impact hammer. Your first thought might be how could you possibly have a problem with missing a mode with 9 references. And wouldn't you know it, that all 9 accelerometers all happened to be all located on the node of one of the modes of the structure. Figure 3 shows this very unbelievable test that was run where, with 9 references, one of the modes of the structure was missed.

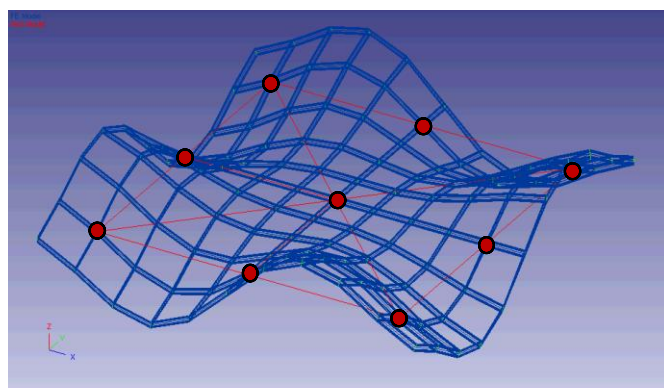


Figure 3 – Plate Mode Missed with 9 References

I hope that this helps to explain that you need to be very careful when identifying the reference for a modal test. If you have any other questions about modal analysis, just ask me.