Illustration by Mike Avitabile

Do you really need to measure FRFs? Or is a Transmissibility OK? We need to discuss this.

So there are many times that transmissibility is made for measurements in many different situations. This might be due to the fact that the data is collected during a shaker qualification test where the test article is mounted on a big shaker and all of the "device under test" accelerometers are measured relative to the base acceleration input to the test article.

Or it may be that the measurements are made on equipment in operation and the force cannot be measured and only response measurements with accelerometers are available. This is common when flight tests, vehicle test, suspension tests or similar tests are performed. This might be the only data that is available. But there are some slight differences that need to be noted. And it is also important to make sure that we are all using the same nomenclature when we use all these fancy words; sometimes I find that the words mean different things in different industries so it is always important to check the definitions are understood.

So let's make some simple definitions to explain some of the differences in all the measurements we may possibly make. If we make the following definitions,

x(t) - time domain input to the system
y(t) - time domain output to the system
Sx(f) - linear Fourier spectrum of x(t)
Sy(f) - linear Fourier spectrum of y(t)
H(f) - system transfer function
h(t) - system impulse response

then Fig 1 shows the input-output schematic for linear spectra.

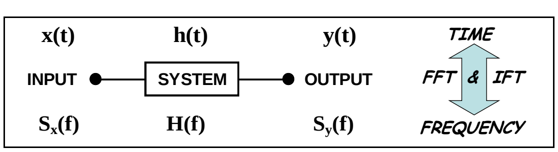

Figure 1 – Linear Spectra

And if we make these additional definitions,

Rxx(t) - autocorrelation of the input signal x(t)
Ryy(t) - autocorrelation of the output signal y(t)
Ryx(t) - cross correlation of y(t) and x(t)

Gxx(f) - autopower spectrum of x(t) $G_{xx}(f)=S_x(f) \bullet S_x^*(f)$
Gyy(f) - autopower spectrum of y(t) $G_{yy}(f)=S_y(f) \bullet S_y^*(f)$
Gyx(f) - cross power spectrum of y(t) and x(t) $G_{yx}(f)=S_y(f) \bullet S_x^*(f)$

then Fig 2 shows the input-output schematic for power spectra.

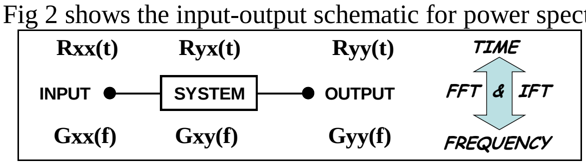

Figure 2 – Power Spectra

So now that we have these equations defined, let's identify some measurements we typically make and understand how they are used to compute things such as the FRF and Transmissibility for instance.

The first thing to notice is that Linear Spectra are complex valued functions that have both magnitude and phase – so S_x and S_y are complex linear spectra. But their companion power spectra, G_{xx} and G_{yy}, are not complex valued but are real valued, magnitude only measurements. This is very important because they have no phase information associated with them. But notice that the cross spectrum, G_{yx}, is a complex valued measurement that has both magnitude and phase.

So let's proceed and identify the FRF and Transmissibility. The FRF is the cross power spectrum divided by the input power spectrum whereas the Transmissibility is just the ratio of the output spectrum divided by the input spectrum. These are given as:

$$H = \frac{S_y \bullet S_x^*}{S_x \bullet S_x^*} = \frac{G_{yx}}{G_{xx}} \qquad TR = \frac{S_y \bullet S_y^*}{S_x \bullet S_x^*} = \frac{G_{yy}}{G_{xx}}$$

So while we generally say that both measure the output relative to the input, there is a big difference – the FRF is a complex function with both magnitude and phase whereas the Transmissibility (TR) is just the ratio of the magnitudes; this is very different because of the lack of phase. But there is one very important difference. Generally the FRF is measured with a reference to a measured force whereas the TR has no force measured as it is typically obtained. This is very important when the data is needed for development of a calibrated model for model validation, structural dynamic modification, system model development, and forced response studies – a measured force is needed to calibrate the model so to speak.

So now that we have all the definitions out of the way, let's look at some measurements for a FRF and TR to show a few differences. A simple free-free beam will be used for some typical measurements. In the first measurement, a drive point FRF is made with an impact hammer and accelerometer. Figure 3 shows the Log Mag and phase for the drive point measurement; notice that the function is complex as shown in the figure. Figure 4 shows the Log Mag and phase for a cross measurement from one end of the beam to the other end of the beam; this measurement is also complex valued.

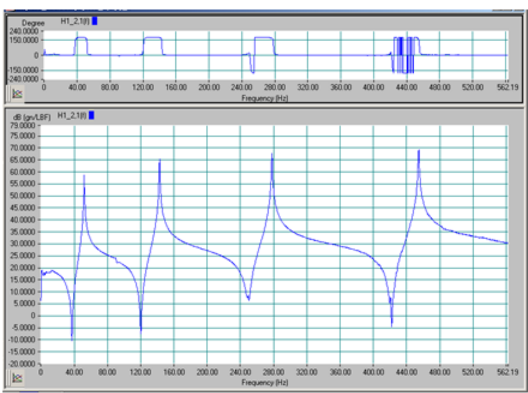
Figure 3 – FRF Drive Point Measurement on Beam

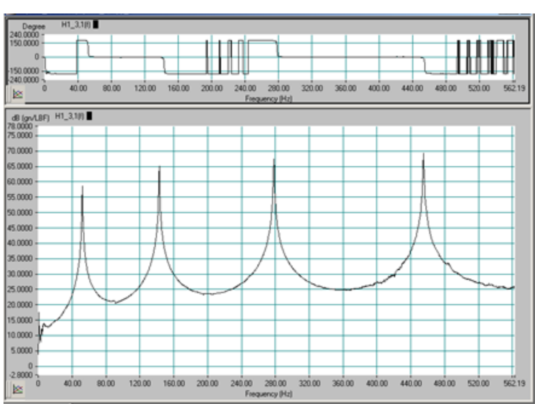
Figure 4 – FRF Cross Measurement on Beam

Figure 5 shows the autopower spectrum of the accelerometer at the drive point measurement location and Figure 6 shows the autopower spectrum of the accelerometer at the cross measurement location.

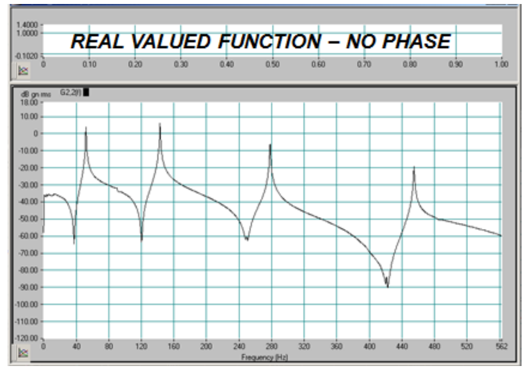

Figure 5 Autopower Spectrum of the Drive Point Location

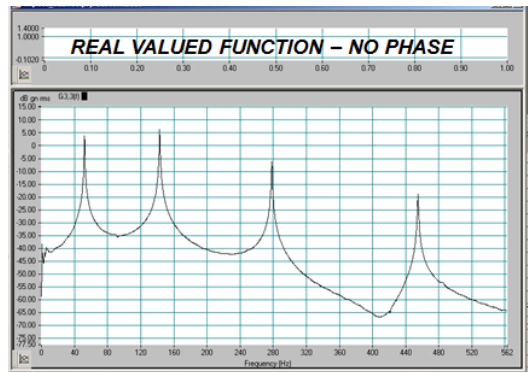

Figure 6 Autopower Spectrum of the Cross Point Location

Now both of the spectrum have some similar features when compared to the FRF measurements shown in Figure 3 and 4. But there is a very important piece of imformation missing which is the phase of the measurement. The power spectra are real valued functions but do not have any phase information. So when relating the magnitudes to each other there is no phase information that can be obtained from the measurement.

In order to have any directional information, there needs to be a complex measurement obtained so that phase is included. Now don't get me wrong here because the transmissibility can be very useful in many cases where no other measurement is possible. But we just have to make sure that we realize that there is some critical information needed if we want to understand the mode shapes of the structure. If you have any other questions about modal analysis, just ask me.