Illustration by Mike Avitabile

When the transfer function is evaluated along the frequency axis, the damping is zero.
Does this mean there is no damping in the system. Let's clarify this confusing point.

Well I find that this becomes a confusing point for many people so let's try to talk about it and explain what is actually happening with this. So I will discuss a few items along the way here as part of the discussion.

First, let's write the system transfer function in partial fraction form

$$h(s) = \frac{a_1}{(s - p_1)} + \frac{a_1^*}{(s - p_1^*)}$$

and realize that the roots or poles of this function for an underdamped system can be written as

$$s_{1,2} = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n{}^2} = -\sigma \pm j\omega_d$$

Because the function is complex, the roots will be a function of two variables, σ and ω , which are the real and imaginary parts of the root. The numerator is called the residue of the system transfer function (and is so named because it comes from the Residue Theorum used to evaluate the function).

Now when we plot this function, the plot is going to map a surface because the function is defined by two independent variables, namely σ and ω . So if we hold σ constant and vary ω and then incrementally change σ and recompute the range of ω there will be a matrix of complex numbers that are generated. Because the numbers are complex, we can make a plot of the real and imaginary parts separately but we could also plot the magnitude and phase for the function. In any event, this surface can be plotted in any one of these forms to describe the system transfer function.

This is shown in Figure 1 (from Vibrant Technology webpage). We can discuss each of the pieces of the system transfer function but I really want to concentrate on the magnitude of this function for the discussion here. (But we need to always remember that this is a complex valued function that has real and imaginary, or magnitude and phase, to describe the total function.)

So when we say that we evaluate the function at $\sigma = 0$, we aren't really saying that the damping is zero but rather that the function is evaluated along the $j\omega$ axis.

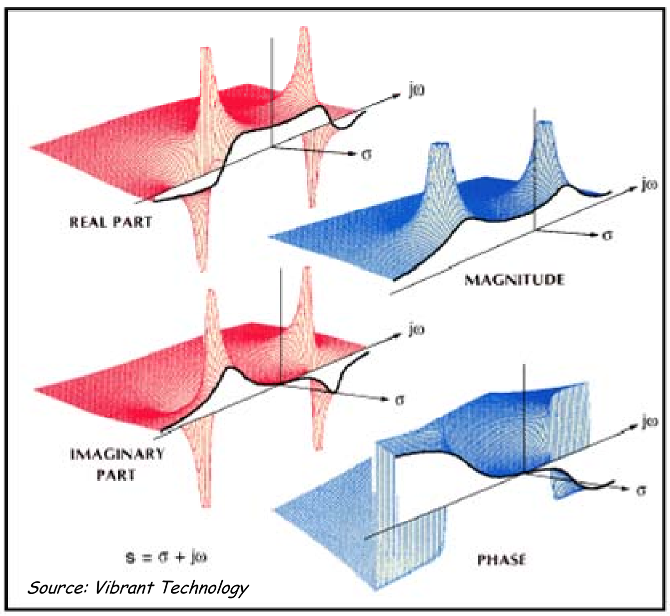

Figure 1 – System Transfer Function

Now if we write this equation evaluated this way then we can write the frequency response function as

$$h(j\omega)=h(s)\ \Big|_{s=j\omega}=\frac{a_1}{(j\omega-p_1)}+\frac{a_1^*}{(j\omega-p_1^*)}$$

And if we were to look at the magnitude of the system transfer function evaluated along the jω axis, and project the face of that cut along that axis we would see the plot shown in Figure 2 that is projected from that slice. This what we measure in the FFT analyzer - the frequency response function. And we can see that there is only one independent variable ω used to describe that function. We would also notice that we only have a line now rather than the surface described for the system transfer function.

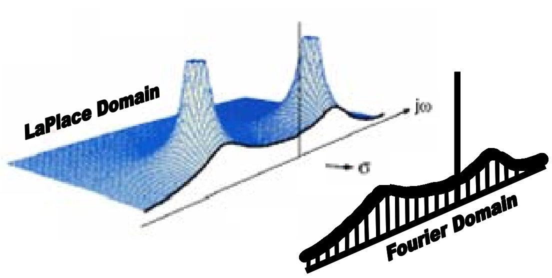

Figure 2 – System Transfer Function (Magnitude) with Frequency Resonse Function

So now we have a handle on where this frequency response function comes from. Now we want to describe the splane and the system transfer function surface. Well, the surface looks like a tent with two poles so I want to use this as an analogy with a wedding with a seating arrangement under the tent. We know that there are two sides to the seating arrangement – the bride and groom (the pole and conjugate of the pole). Now you could be seated on either side depending on which side of the wedding party you are with.

Let's say that you are with the bride's side of the wedding and you are seated in the first row-second seat. Now when you sit down and look up you will notice that the tent is a certain height above your head (the magnitude at that particular value of σ and ω). You will also notice that there is a mirror image seat on the groom's side (conjugate) at the first row-second seat; and the height above that seat is the same in terms of its magnitude.

But let's say that someone else was seated at the second row-third seat on the bride's side. Now at that point, the height of the tent is much higher than the first case. And of course, there is also a mirror image seat on the groom's side which has the same height.

So each of these seats has a particulat tent height above the seat location. That height maps the surface of the tent. But you notice that there is one seat on both sides that corresponds to where the pole is located; these are analgous to the roots which are complex conjugate pairs. Notice that no one can sit there and no one can really tell what the magnitude of the tent is at that location because it is undefined; the magnitude of the system transfer function can not be determined at the pole (root) of the tent because it is undefined at that location.

So this tent analogy is a pretty good description of the system transfer function. The value of the function is determined by the location in the seating plan. The amplitude changes depending on location. The system transfer function is undefined at the poles or roots of the system; that is where the poles of the tent are located and no one can sit at that location to determine the amplitude or height of the tent. (We use the Residue Theorum to evaluate this.)

And of course we all know that the first row is the most important row. In fact, that is where Mr Fourier resides – right along the jω axis – which is the slice we took out of the system transfer function.

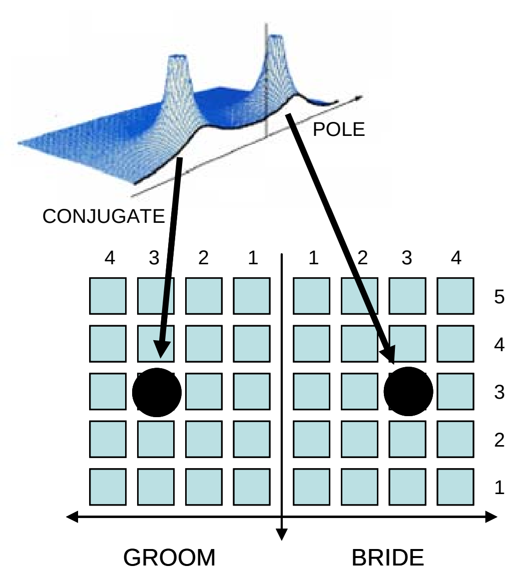

Figure 3 – The S-Plane Representation

So I hope these simple examples shed a little more light on the system transfer function and the frequency response function and how they are related to each other. One last note regarding a recent wedding where the best man was asked to introduce the bridal party. He did just fine up until he introduced (for the first time) the groom and bride as John and Angela – it was also the last time because the bride's name wasn't Angela! But that's another story.

If you have any other questions about modal analysis, just ask me.