MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

Illustration by Mike Avitabile

Do I need to have an accelerometer mounted in the X, Y and Z directions to do a modal test? Well ... let's discuss this.

This is an item that often causes confusion for many people. There is some preconceived notion that there must be an accelerometer mounted in each of the three principal directions in order to acquire data for a modal test. Well, it turns out that this is not necessary but in some tests it may be strongly advised or even required. But many times people think that you can't get three dimensional mode shapes unless you have accelerometers in all three directions.

The basic equation we use for estimating parameters can be written in one form as

$$\left[H(s)\right] = \text{lower residuals} + \sum_{k=i}^{j} \frac{\left[A_k\right]}{\left(s - s_k\right)} + \frac{\left[A_k^*\right]}{\left(s - s_k^*\right)} \quad + \text{upper residuals}$$

The terms in the matrix, [A], are the residues which are obtained from the curvefitting process; we also get the poles, or frequency and damping, from the denominator of the equation. But these residues are directly related to the mode shapes from

$$\left[A(s)\right]_k = q_k \left\{u_k\right\} \left\{u_k\right\}^T$$

This relationship between the residues and the mode shapes holds the answer to the question posed. Let's expand that equation to look at some of the terms that are found in each term of the matrix.

$$\begin{bmatrix} a_{11k} & a_{12k} & a_{13k} & \cdots \\ a_{21k} & a_{22k} & a_{23k} & \cdots \\ a_{31k} & a_{32k} & a_{33k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = q_k \begin{bmatrix} u_{1k}u_{1k} & u_{1k}u_{2k} & u_{1k}u_{3k} & \cdots \\ u_{2k}u_{1k} & u_{2k}u_{2k} & u_{2k}u_{3k} & \cdots \\ u_{3k}u_{1k} & u_{3k}u_{2k} & u_{3k}u_{3k} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

And if we were to look at each of the columns we would see the mode shape is contained in the column with some scalar multipliers; we would also see that due to reciprocity, the rows also contain the mode shapes. If we were to look at one column, such as the first column, then we would see

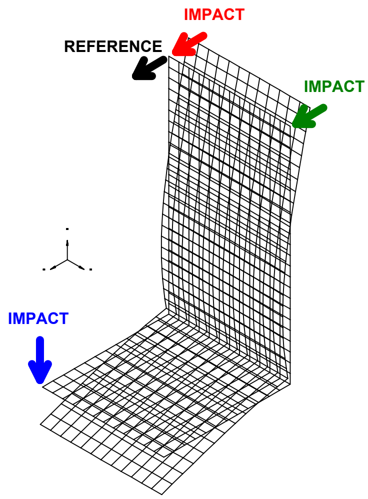
$$\begin{Bmatrix} a_{11k} \\ a_{21k} \\ a_{31k} \\ \vdots \end{Bmatrix} = q_k u_{1k} \begin{Bmatrix} u_{1k} \\ u_{2k} \\ u_{3k} \\ \vdots \end{Bmatrix}$$

So the value of the mode shape that is factored out of the equation is called the "reference" DOF. In other words, all of the measurements are affected by the value of this reference DOF. If this DOF is zero (at the node of the mode) then no matter how many measurements are made, that particular mode will not be observed from the measured data.

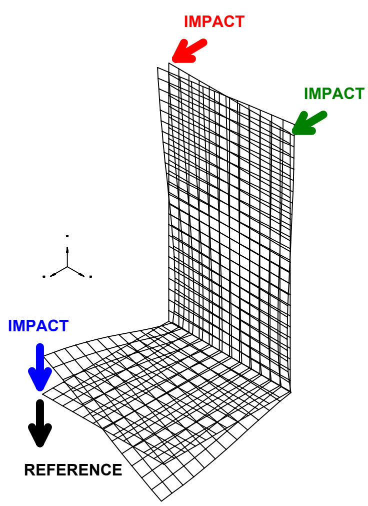
It is this basic equation that really contains the answer to the question raised. As long as the reference DOF has a non-zero value for each of the modes of interest, then the frequency response function will have residue associated with that input-output relationship. As long as the mode shape in the X, Y and Z direction has a value associated relative to the reference DOF, then the mode shape(s) can be observed from measurements made relative to that reference DOF. It's that simple!

Now let's use a simple structure to illustrate this point. A simple L-shaped bracket will be used for discussion purposes and illustration of the reference DOF and its relationship to all of the measured DOFs for the experimental modal survey. For the discussion, the reference point where the accelerometer is to be mounted will be shown in black in the following figures and the various impact locations will be shown in blue, red and green for distinction between the different points.

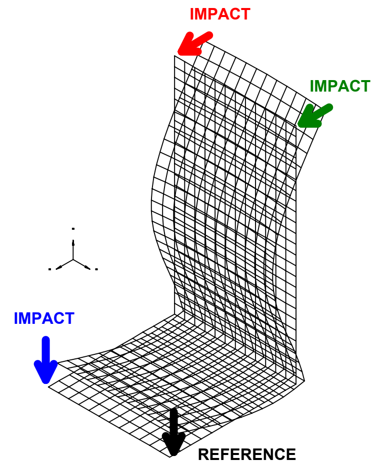
For this first mode of this bracket, the reference accelerometer could possibly be located at the upper corner of the structure in the x-direction. Notice that if the structure is impacted at the upper corners of the bracket in the x-direction (red or green) or on the lower corner in the z-direction (blue), the structure has significant response at all these locations. This implies that if the structure is impacted in the x-direction (red or green) in the upper corner, there is response in the x-direction at the reference point. And if the structure is impacted on the lower corner (blue) in the z-direction, there is response at the reference point in the x-direction. So this mode can easily be seen from the selected reference location.



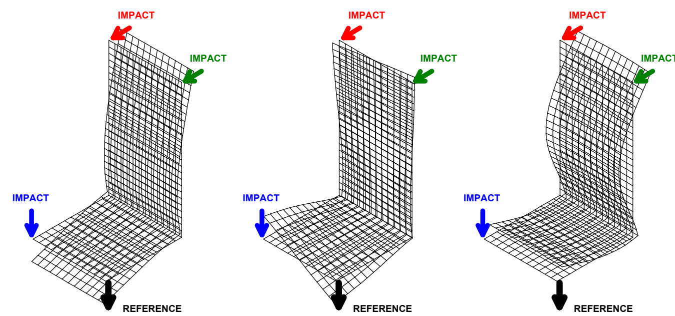
For the second mode of the structure, the reference accelerometer could possibly be located at the lower corner on the structure in the z-direction. If the structure is impacted in the z-direction at the same point (blue), the structure has significant response at this point for this mode. But also notice that if the structure is impacted at the upper corner (red and green) in the x-direction, there is response at the reference accelerometer location in the z-direction. So this reference is good for this mode.



And if the third mode is considered with the reference accelerometer located at the lower corner in the z-direction, all three impact locations on the structure have significant response at this point for this mode.



Now the real question is if there is ONE reference location that can be selected that will adequately capture the dynamic characteristics of the structure for all the modes of interest. For this case, it seems reasonable that the lower corner of the structure in the z-direction is sufficient to observe all of the modes for this case.



Now we can see that the single reference point is sufficient to adequacy observe all the modes of interest - and only one direction is necessary to accomplish this. Of course, if more references are used this is totally acceptable and is definitely a better way to test the structure - but these extra references are not necessarily needed in order to extract modes shapes that are three dimensional in nature.

I hope that this helps to clear up the misconception regarding the need to measure in three separate reference directions for a modal test. Think about it and if you have any more questions about modal analysis, just ask me.