MODAL SPACE - IN OUR OWN LITTLE WORLD

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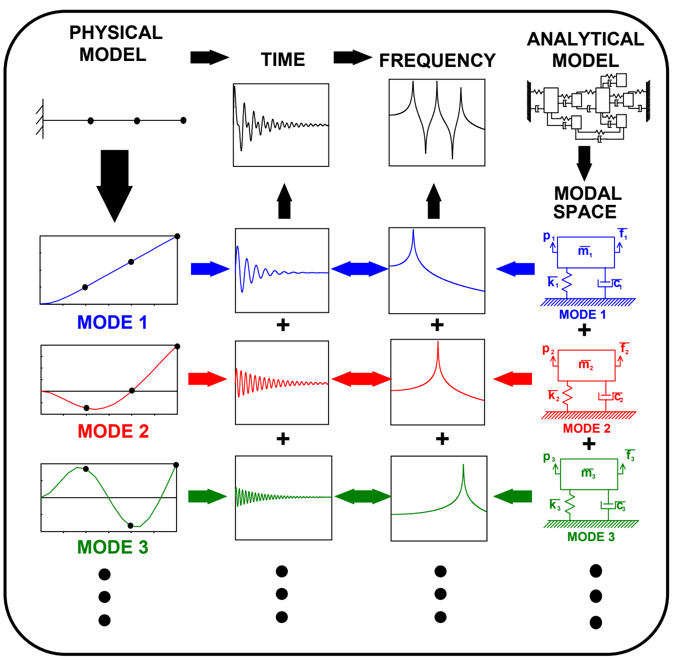
Could you explain the difference between time domain, frequency domain and modal space?
I hear it all the time but I'm not sure what's the difference.
There's a lot to explain but let's start with something simple.

This question gets asked often. There's a lot of different aspects relating to this so let's start with a simple explanation without using too much math and explain all of this with a simple schematic. Let's use the figure to discuss all these different aspects of the time domain, frequency domain, modal space and physical space. Now there are a lot of parts to discuss in the figure, so let's take them in pieces - one at a time - and then summarize everything at the end. You might also want to remember the discussion we had before when you asked me about what modal analysis was all about ("Could you explain modal analysis for me?") to help with the discussion here.



First, let's consider a simple cantilever beam and imagine that the beam is excited by a pulse at the tip of the beam. The response at the tip of the beam will contain the response of all the modes of the system (shown in the black time response plot); notice that there appears to be response at several different frequencies. This time response at the tip of the beam can be converted to the frequency domain by performing a Fourier Transform of the time signal. There is a significant amount of math that goes along with this process but it is a common transformation that we perform all the time. The frequency domain representation of this converted time signal is often referred to as the frequency response function, or FRF for short (shown in the black frequency plot); notice that there are peaks in this plot which correspond to the natural frequencies of the system.

Before we discuss the time and frequency plots any further, let's talk about the physical model in the upper left part of the figure. We know that the cantilever beam will have many natural frequencies of vibration. At each of these natural frequencies, the structural deformation will take on a very definite pattern, called a mode shape, as described previously [1]. For this beam, we see that there is a first bending mode shown in blue, a second bending mode shown in red and a third bending mode shown in green. Of course, there are also other higher modes not shown and we will only discuss the first three modes here but it could easily be extended to higher modes.

Now the physical beam could also be evaluated using an analytical lumped mass model or finite element model (shown in black) in the upper right part of the figure. This model will generally be evaluated using some set of equations where there is an interrelationship, or coupling, between the different points, or degrees of freedom (dof), used to model the structure. This means that if you pull on one of the dofs in the model, the other dofs are also affected and also move. This coupling means that the equations are more complicated in order to determine how the system behaves. As the number of equations used to describe the system get larger and larger, the complication in the equations becomes more involved. We often use matrices to help organize all of the equations of motion describing how the system behaves which looks like

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\}$$

where [M], [C], [K] are the mass, damping and stiffness matrices respectively, along with the corresponding acceleration, velocity and displacement and the force applied to the system. Usually the mass is a diagonal matrix and the damping and stiffness matrices are symmetric with off-diagonal terms indicating the degree of coupling between the different equations or dofs describing the system. The size of the matrices is dependent on the number of equations that we use to describe our system. Mathematically, we perform something called an eigensolution and use the modal transformation equation to convert these coupled equations into a set of uncoupled single dof systems described by diagonal matrices of modal mass, modal damping and modal stiffness in a new coordinate system called modal space described as

$$\begin{bmatrix} \diagdown & & \\ & \overline{M} & \\ & & \diagdown \end{bmatrix}\{\ddot{p}\}+\begin{bmatrix} \diagdown & & \\ & \overline{C} & \\ & & \diagdown \end{bmatrix}\{\dot{p}\}+\begin{bmatrix} \diagdown & & \\ & \overline{K} & \\ & & \diagdown \end{bmatrix}\{p\}=[U]^T\{F\}$$

So we can see that the transformation from physical space to modal space using the modal transformation equation is a process whereby we convert a complicated set of coupled physical equations into a set of simple uncoupled single dof systems. And we see in the figure that the analytical model can be broken down into a set of single dof systems where the single dof describing mode 1 is shown in blue, mode 2 is shown in red and mode 3 is shown in green. Modal space allows us to describe the system easily using simple single dof systems.

Now let's go back to the time and frequency responses shown in black. We know that the total response can be obtained from the contribution of each of the modes. The total response shown in black comes from the summation of the effects of the response of the model shown in blue for mode 1, red for mode 2 and green for mode 3. This applies whether I describe the system in the time domain or the frequency domain. Each domain is equivalent and just presents the data from a different viewpoint. It's a lot like money - as I go from country to country, the money in each country looks different but it's really the same thing. So we can see that the total time response is made up of the part of the time response due to the contribution of the time response of mode 1 shown in blue, mode 2 in red and mode 3 in green. We can also see that the total FRF is made up of the part of the FRF due to the contribution of the FRF of mode 1 shown in blue, mode 2 in red and mode 3 in green. (We have only shown the magnitude part of the FRF here; this function is actually complex which is correctly displayed using both magnitude and phase or real and imaginary parts of the FRF).

Since we can break the analytical model up into a set of single dof systems, we could determine the FRF for each of the single dof systems as shown with mode 1 in blue, mode 2 in red, and mode 3 in green. We could also determine the time response for each of these single dof systems through a closed form solution for the response of a single dof system due to the pulse input or we could simply inverse Fourier Transform the FRF for each of the single dof systems. We could also measure the response of the beam at the tip due to the pulse and filter the response of each of the modes of the system, and we would see the response of each of the modes of the system with mode 1 shown in blue, mode 2 in red and mode 3 in green. (Of course, I'm simplifying a lot of theory here so we can understand the concepts.)

Now that we have pulled apart all the pieces of the figure, I think it should be much clearer that there is really no difference between the time domain, frequency domain, modal space and physical space. Each domain is just a convenient way for presenting or viewing data. However, sometimes one domain is much easier to see things than another domain. For instance, the total time response does not clearly identify how many modes there are contributing to the response of the beam. But the total FRF in the frequency domain is much clearer in showing how many modes are activated and the frequency of each of the modes. So often we transform from one domain to another domain simply because the data is much easier to interpret.

While there is a lot more to it all, I hope this simple schematic and explanation helps to put everything in better perspective. Think about it and if you have any more questions about modal analysis, just ask me.