MODAL SPACE - IN OUR OWN LITTLE WORLD

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Why is calibration and mode shape scaling important?
And does it make a difference?
Let's talk about this

Calibration and mode shape scaling are two important items for the development of an accurate dynamic model that would be used for other structural dynamic studies. Some of these would be simulation and prediction, modification, correlation, to name a few. While there may be some instances when calibration and scaling may not be critical, I will always recommend that they are done since this may be the only data ever acquired. First let's discuss calibration and then discuss scaling.

Calibration of the whole acquisition system is very important. Back in the early days of 2 channel FFT analyzers, many times we may have stepped around calibration when performing troubleshooting or quick investigative tests since we were only interested in the ratio of output to input - so the exact units may not have been critical. This may have been tolerable since we may have only been interested in general shapes of the structure. But as soon as the use of the modal data for simulation, prediction, etc. was needed, then a fully calibrated model was necessary. An accurate calibration was required when the dynamic model is used for other structural dynamic studies.

As larger channel FFT analyzers became available, the use of several accelerometers (possibly with different sensitivities) required at least some nominal calibration value to be used. If not, then different regions of the structure could possibly show relative differences in shape which could cause confusion in understanding the mode shapes.

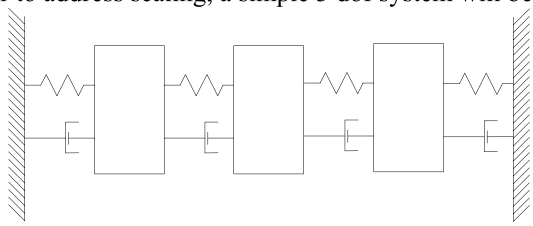
So what calibration should be performed? Well a complete calibration is always best. This would involve a complete calibration of an entire acquisition channel as a unit - the accelerometer, signal conditioner, ADC channel together.

While calibration of each individual piece is often acceptable, the calibration of the complete system together is preferred. There are a variety of types of calibration. Accelerometers can be calibrated relative to some well-maintained reference accelerometer that is traceable to a source. This can be performed in the lab using a piggyback arrangement for the test accelerometer to the reference accelerometer. Or the test accelerometer can be calibrated through drop test with some known mass. Another common calibration utilizes an excitation through a force gage mounted to a known mass with an accelerometer. This ensures that the ratio of force to acceleration using the equation of motion with the known mass. (The individual transducers can be identified if one of them is known). The most accurate way to perform this calibration is through the acquisition channels to be used for the test.

And while many calibration service companies provide calibration in fixed increments (ie, 50, 100, 200, 500, 1000, ...), this only provides information at those discrete frequencies. The better way of calibrating is to perform broadband input excitation over the frequency range of interest.

Now that calibrated modal data has been addressed, we need to discuss mode shape scaling. Yes - I know that the shape is the relative motion between points. But there is a scaling that needs to be preserved. That is, the relationship between the modal mass, modal damping and modal stiffness. The shapes can be anything but the relationship between the shapes and physical quantities is very important. The shapes can be scaled to anything you'd like - but the most common is *unit modal mass* scaling (but others such as unit length, largest value equals one are also common). The most important item is that the shapes are scaled to some quantity that is identified for future reference. Scaling can be a critical item with respect to further

use of the dynamic model for simulation, prediction, correlation, etc.
In order to address scaling, a simple 3 dof system will be used.



The equation of motion and specific system values are:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[F(t)]$$

$$\begin{bmatrix}1&&\\&1&\\&&1\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{Bmatrix}+\begin{bmatrix}0.2&-0.1&\\-0.1&0.2&-0.1\\&-0.1&0.2\end{bmatrix}\begin{Bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{Bmatrix}+\begin{bmatrix}20000&-10000&\\-10000&20000&-10000\\&-10000&20000\end{bmatrix}\begin{Bmatrix}x_1\\x_2\\x_3\end{Bmatrix}=\begin{Bmatrix}f_1\\f_2\\f_3\end{Bmatrix}$$

and the corresponding eigensolution yields:

$$[[K]-\lambda[M]]\{X\}=\{0\}$$

$$\left[\Omega^2\right]=\begin{bmatrix}5858&&\\&20000&\\&&34142\end{bmatrix}\;;\;[U]=\left[\{u_1\}\;\{u_2\}\;\{u_3\}\right]=\left[\begin{Bmatrix}0.500\\0.707\\0.500\end{Bmatrix}\begin{Bmatrix}0.707\\0\\-0.707\end{Bmatrix}\begin{Bmatrix}-0.500\\0.707\\-0.500\end{Bmatrix}\right]$$

Now the first mode of the system will be the only one addressed.
The frequency, damping and complex pole for mode 1 are:

Frequency 12.18Hz Damping 0.038%
Complex pole -0.029 ± j 76.537 rad/sec

Let's recall that the poles and residues are the values that describe the FRF measured. For mode 1, this is

$$h(j\omega)=h(s)\Big|_{s=j\omega}=\frac{a_1}{(j\omega-p_1)}+\frac{a_1^*}{(j\omega-p_1^*)}$$

Now let's also recall that the residues are directly related to the mode shapes of the system from

$$\left[A(s)\right]_k=q_k\,\{u_k\}\,\{u_k\}^T$$

which can be expanded as

$$\begin{bmatrix}a_{11k}&a_{12k}&a_{13k}&\cdots\\a_{21k}&a_{22k}&a_{23k}&\cdots\\a_{31k}&a_{32k}&a_{33k}&\cdots\\\vdots&\vdots&\vdots&\ddots\end{bmatrix}=q_k\begin{bmatrix}u_{1k}u_{1k}&u_{1k}u_{2k}&u_{1k}u_{3k}&\cdots\\u_{2k}u_{1k}&u_{2k}u_{2k}&u_{2k}u_{3k}&\cdots\\u_{3k}u_{1k}&u_{3k}u_{2k}&u_{3k}u_{3k}&\cdots\\\vdots&\vdots&\vdots&\ddots\end{bmatrix}$$

The scaling constant 'q' is a very important term in this equation. While there are many different types of scaling that may be used, the most common one is *unit modal mass* scaling. (This is also a very common scaling used in finite element modeling software packages). With this, the system parameters of modal mass, modal damping and modal stiffness are defined as:

modal mass $\quad \overline{m}_k=\dfrac{1}{q_k\overline{\omega}_k}$

modal damping $\quad \overline{c}_k=2\sigma_k\overline{m}_k$

modal stiffness $\quad \overline{k}_k=(\sigma_k^2+\overline{\omega}_k^2)\overline{m}_k$

Now if we consider the first column of these equations, then the residues can be related to the mode shapes using

$$\begin{Bmatrix}a_{11k}\\a_{21k}\\a_{31k}\\\vdots\end{Bmatrix}=q_k u_{1k}\begin{Bmatrix}u_{1k}\\u_{2k}\\u_{3k}\\\vdots\end{Bmatrix}$$

We notice that there is a scale factor 'q' which is important in this equation. This scaling constant helps to preserve the proper scaling relationship between the mode shapes and the system modal mass, modal damping and modal stiffness. Notice that if we take a measurement such as h_{31} that a_{31}=q u_3u_1 (also h_{21} that a_{21}=q u_2u_1 and so on). For each of these equations, there is one extracted value of the residue (from the curvefitting process) but two values of the mode shape. So the best that can be said about the mode shape is that there is a 'relative' motion between the various points. This relative motion using the residues can be animated and provides a wealth of knowledge. However, if the drive point measurement is considered, then we see that h_{11} provides a_{11}=q u_1u_1 and it is this equation that can be used to solve for u_1 which is then used to scale all the other terms that were measured.

Every now and then I will hear someone say that there is no need to scale the mode shapes and that there is no need to take a drive point measurement. While this is true to in order to visually observe the mode shapes, without any scale factor, this modal information cannot be used for any further analytical manipulation using this data. The scaled modal data is required for any further analyses such as structural modification, forced response, prediction, simulations, correlation, etc.

Since the modes in this example are real normal modes, the residues are complex valued but will only have an imaginary part of the residue. In order to simply the numbers, the residue will be converted to a real valued expression using r = 2 j a - (note that this is a common representation of the residue in many commercially available modal analysis packages).

The values of the residues 'r' for this example for mode 1 are

dof1 = (0.003266 ± j 0.0)
dof2 = (0.004619 ± j 0.0)
dof3 = (0.003266 ± j 0.0) .

Then the relationship of the residues to the mode shapes with the scaling factor are given as:

$$\begin{Bmatrix}0.32664E-2\\0.46194E-2\\0.32664E-2\end{Bmatrix}=\begin{Bmatrix}r_{11}\\r_{21}\\r_{31}\end{Bmatrix}^{(1)}=q_1u_1\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}^{(1)}=\frac{1}{76.537}(0.500)\begin{Bmatrix}0.500\\0.707\\0.500\end{Bmatrix}$$

So it can be seen that the is a definite scaling relationship that does exist and the mode shapes ***must*** be scaled using the drive point measurement in order to accomplish this. If you have any other questions about modal analysis, just ask me.