MODAL SPACE - IN OUR OWN LITTLE WORLD

by Pete Avitabile

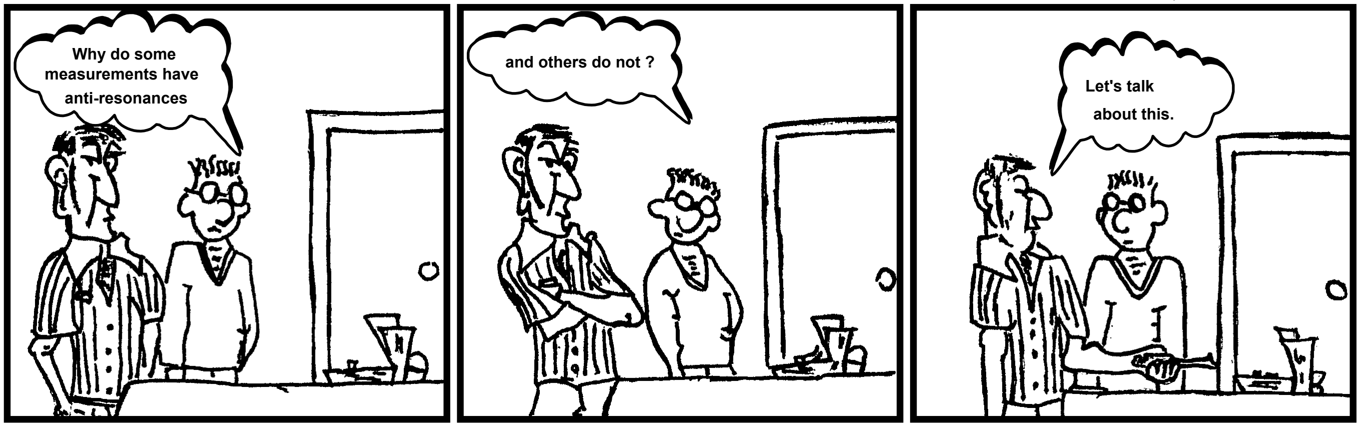
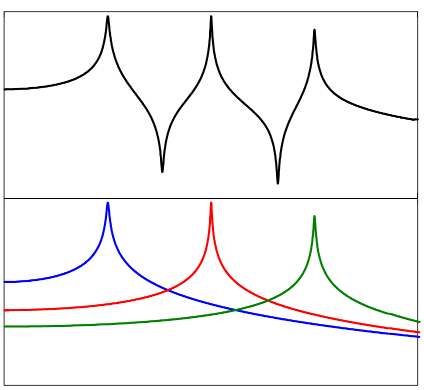


Illustration by Mike Avitabile

Why do some measurements have anti-resonances and others do not?
Let's talk about this.

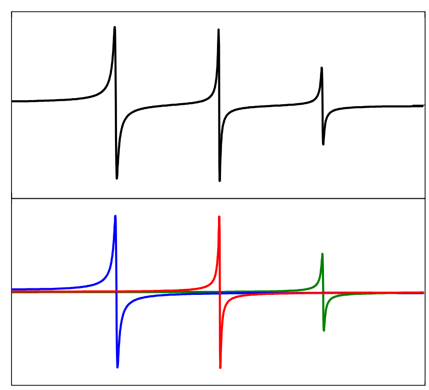
This is a good question. You are absolutely correct - some measurements have anti-resonances and others do not. But why does this happen. Let's first discuss some properties of a particular measurement called a drive point measurement and then extend this discussion to explain how anti-resonances occur in a measurement.

Let's first explain a drive point measurement. A drive point measurement is one where the input force and output response are made at the same point and in the same direction. A typical drive point measurement is shown in Figure 1.

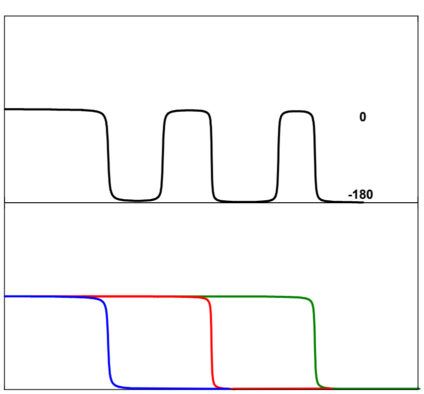


MAGNITUDE

$$h_{ij}(j\omega) = \frac{a_{ij1}}{(j\omega - p_1)} + \frac{a^*_{ij1}}{(j\omega - p^*_1)} + \frac{a_{ij2}}{(j\omega - p_2)} + \frac{a^*_{ij2}}{(j\omega - p^*_2)} + \frac{a_{ij3}}{(j\omega - p_3)} + \frac{a^*_{ij3}}{(j\omega - p^*_3)}$$

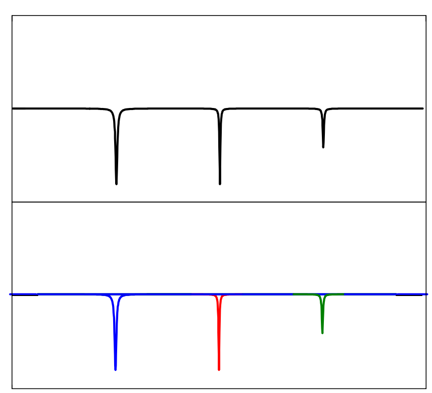


REAL



PHASE

$$h_{ij}(j\omega) = \frac{q_1 u_{i1} u_{j1}}{(j\omega - p_1)} + \frac{(q_1 u_{i1} u_{j1})^*}{(j\omega - p^*_1)} + \frac{q_2 u_{i2} u_{j2}}{(j\omega - p_2)} + \frac{(q_2 u_{i2} u_{j2})^*}{(j\omega - p^*_2)} + \frac{q_3 u_{i3} u_{j3}}{(j\omega - p_3)} + \frac{(q_3 u_{i3} u_{j3})^*}{(j\omega - p^*_3)}$$



IMAGINARY

Figure 1 - Drive Point FRF (Magnitude, Phase, Real, Imaginary)

For a driving point measurement several items can be noted:

- all resonances are separated by antiresonances as seen in the magnitude plot
- the phase looses 180 degrees of phase passing over a resonance and gain 180 degrees of phase passing over an antiresonance
- the peaks in the imaginary part of the FRF must all point in the same direction

The drive point measurement can be viewed as a summation of all the modes or as the contribution due to each mode. As seen in the four plots in Figure 1, the upper plot contains the summation due to all the modes and the lower plot shows the contribution due to each mode. For the first three modes shown, the frequency response function is made up of the sum of each of the single degree of freedom oscillators describing each mode of the system. For reference, recall that the frequency response function equation can be written as either residues or mode shapes as shown in Figure 1.

Now that the drive point measurement is understood, several other items can be discussed. For instance, the imaginary part of the frequency response function must all have the same direction and in this condition an anti-resonance exists between each mode. This is due to the fact that the magnitude of the FRF of mode 1 and mode 2 is equal at the anti-resonant frequency. But at this frequency, while the magnitudes are equal, the phase is 180 degrees out of phase with each other. This implies that the sum of mode 1 and mode 2 are equal and opposite. Therefore the function trends towards zero. (There is actually a contribution from other modes that is generally very small when the modes are far spaced as shown.)

Now this implies that when the imaginary part of each mode has an opposite sign, the phase is not necessarily out of phase - and then the modes add and an anti-resonance does not result. So each measurement can have anti-resonances or no anti-resonances (saddles) depending on the direction of the imaginary part of the frequency response function. When the imaginary part of the frequency response function for sequential modes have the same direction, then an anti-resonance will exist between those two modes. When the imaginary part of sequential modes have different signs or directions, then a saddle exists between those two modes.

Actually, the direction (or sign) of the function is directly related to the mode shapes of the system. As seen in Figure 1, the frequency response function can be written in the form of residues. But the residues can be expressed in terms of the mode shapes of the system. When written as mode shapes, the directional sign of the residue can be clearly seen as a result of the mode shape of the system. Figure 2 shows the measurements for a simple 3 DOF system. Upon reviewing each of the individual FRF measurements, the phase relationship and occurrence of anti-resonances and saddles in the frequency response function can now be better understood.

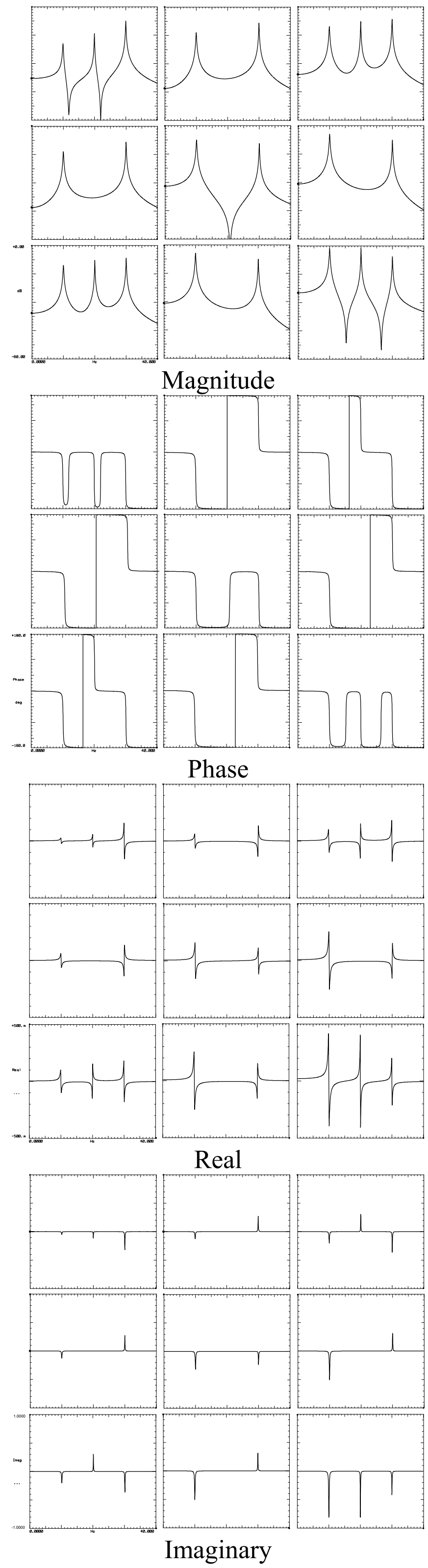


Figure 2 - FRF Matrix for a 3 DOF System

I hope this explanation answers your question. If you have any more questions on modal analysis, just ask me.